# Topological defects in flat nanomagnets: The magnetostatic limit

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We discuss elementary topological defects in soft magnetic nanoparticles in the thin-film geometry. In the limit dominated by magnetostatic forces the low-energy defects are vortices (winding number $n=+1$), cross ties ($n=-1$), and edge defects with $n=-1/2$. We obtain topological constraints on the possible composition of domain walls. The simplest domain wall in this regime is composed of two $-1/2$ edge defects and a vortex, in accordance with observations and numerics. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2168439]

Nanorings made out of a soft ferromagnetic material generate considerable interest as prospective building blocks for nonvolatile random-access memory.[1] An attractive feature of the ring geometry is the existence of two lowest-energy states in which magnetization points in the azimuthal direction (clockwise or counterclockwise) preventing the straying of magnetic field. The switching between the two states can be accomplished by applying the magnetic field in the plane of the ring or by injecting electric current. In both cases the switching is accomplished by nucleating a small bubble of the opposite domain and letting it expand until it occupies the entire ring.[2] Alternatively one can view the process as the creation, propagation, and mutual annihilation of two *domain walls* separating the domains with clockwise and counterclockwise magnetizations. These considerations motivate us to study the properties of domain walls in nanorings.

Domain walls in magnetic nanoparticles differ substantially from domain walls in macroscopically large magnets. The main reason for that is the more prominent role of the surface in smaller samples. Qualitative changes are expected when one (or more) of the particle dimensions crosses a length scale characterizing the strength of ferromagnetic exchange relative to that of the stray field, $\lambda=\sqrt{A/\mu_0 M_0^2}$, or material anisotropy, $\lambda_a=\sqrt{A/K}$. Here $A$ is the exchange constant, $M_0$ is the equilibrium magnetization, and $K$ is the anisotropy constant of the material.[3] The anisotropy scale $\lambda_a$ is particularly large in soft materials and can be considered infinite for submicron particles. The exchange length $\lambda$ is in the range of a few nanometers.

Previous experimental and numerical studies of domain walls in submicron rings[4] demonstrate that the ring curvature does not have a significant impact on the properties of domain walls. We therefore discuss the simpler geometry of a strip. Domain walls in strips were studied numerically by McMichael and Donahue[5] who found (at least) two different types: "transverse walls" in extremely thin and narrow strips and "vortex walls" in thicker and wider ones. Two of us[6] have previously shown that the transverse walls are *composite* objects made of two elementary topological defects located at the opposite edges of the strip. In the limit of an extremely thin and (reasonably) narrow strip, the energy of a domain wall comes mostly from the exchange interaction, so that the magnet is described by the two-dimensional *XY* model with an anisotropy at the edge.[7] In this limit, the elementary topological defects are (a) vortices and antivortices in the bulk of the strip carrying winding numbers of +1 and −1, respectively, and (b) half vortices or "boundary vortices,"[7] confined to the edge and carrying *fractional* winding numbers of ±1/2. A transverse domain wall consists of two half vortices with opposite winding numbers.[6]

In this paper and its companion[8] we discuss the structure and energetics of domain walls in the limit dominated by the magnetostatic interaction, achieved in strips whose width and thickness substantially exceed the exchange length $\lambda$. We demonstrate that the vortex walls observed in this limit are also composite objects containing *three* elementary topological defects: a vortex (winding number of +1) residing between two edge defects (winding number of −1/2). The identification of the elementary topological defects and implications for the composition of domain walls is the subject of this paper. The companion paper[8] deals with the energetics of composite domain walls.

Vortex walls are stabilized when both the width and thickness of a strip substantially exceed the exchange length $\lambda$.[5] In this limit the magnetostatic energy is the dominant contribution to the energy of a domain wall[9,10] and the primary force determining the shape of topological defects. Because the magnetostatic energy is a nonlocal functional of magnetization,[3] energy minimization is a computationally difficult problem. Therefore identification of topological defects is not as straightforward as in the limit dominated by exchange.[6] Furthermore, the magnetostatic energy has a large number of absolute minima and one must search among these solutions for one with the lowest exchange energy, making this a degenerate perturbation problem.

For simplicity we will use the geometry of a thin film with a constant thickness $t$ that is small in comparison to the width of the strip $w$. In this case the shape anisotropy forces the magnetization $\mathbf{M}$ to lie in the plane of the film (with the possible exception of vortex cores[11]). It will be further assumed that the magnetization depends on the coordinates in the plane of the film only,

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$$\mathbf{M} = \mathbf{M}(x,y) = (M_0 \cos\theta, M_0 \sin\theta, 0). \tag{1}$$

For a given configuration of magnetization $\mathbf{M}(\mathbf{r})$ its magnetostatic energy $(\mu_0/2)\int H^2 dV$ can be recast as the Coulomb energy of magnetic charges with density $\rho_m(\mathbf{r}) = -\nabla\cdot\mathbf{M} = M_0(\sin\theta\partial_x\theta - \cos\theta\partial_y\theta)$. Being positive definite, the magnetostatic energy has an absolute minimum of zero, which corresponds to the complete absence of magnetic charges. Thus it makes sense to look for low-energy states with topological defects among configurations with zero charge density in the bulk, $-\nabla\cdot\mathbf{M}=0$, and on the surfaces, $\hat{\mathbf{n}}\cdot\mathbf{M}=0$ (here $\hat{\mathbf{n}}$ is the surface normal). A method for constructing such solutions has been discussed by van den Berg.[12] It yields configurations with domains of smoothly varying magnetization separated by discontinuities in the form of Néel-type domain walls. The walls acquire a finite width when the exchange interaction is taken into account.

The simplest nontrivial example is the configuration with a single vortex at the origin, $\exp[i\theta(x,y)] = \pm i(x+iy)/|x+iy|$. The two signs give two different values of *chirality* (direction of circulation) of the vortex. In both cases the topological charge, or the *winding number*,[13] is +1. This solution has zero density of magnetic charge and thus minimizes the magnetostatic term. Furthermore, it also represents a local minimum of the exchange energy. Taken together, these two observations show that the vortex is a stable configuration. Its energy diverges logarithmically with the system size $R$:

$$E_{+1} \sim 2\pi A t \log(R/\lambda) + E_{\mathrm{core}}. \tag{2}$$

In contrast, the antivortex configuration minimizing exchange energy, $\exp[i\theta(x,y)] = \pm i(x-iy)/|x-iy|$, has a nonzero density of magnetic charge and thus represents a poor starting point for constructing a bulk topological defect with the winding number of $-1$ in this limit. Minimization of the magnetostatic energy in this topological sector is achieved in the configuration known as the cross tie,[14] an intersection of two 90° Néel walls normal to each other (Fig. 1). The energy of an antivortex grows linearly with the length of the Néel walls $L$ emanating from it:

$$E_{-1} \sim \sigma t L + E_{\mathrm{core}} \tag{3}$$

(the core energy is generally different from that of a vortex). The surface tension of the wall $\sigma$ is determined by the competition of exchange and magnetostatic forces. When the film thickness $t$ exceeds the Néel-wall width (of order $\lambda$), the calculation simplifies: the magnetization depends only on the coordinate transverse to the wall. The surface tension is then found by minimizing the total energy per unit area,[3]

$$\sigma = \int dx[A(d\theta/dx)^2 + \mu_0 M_0^2(\cos\theta - \cos\theta_0)^2/2], \tag{4}$$

subject to the boundary conditions $\theta(\pm\infty) = \pm\theta_0$, where $2\theta_0$ is the angle of spin rotation across the wall. Minimization of the total energy yields a domain wall of a characteristic width $\lambda\sqrt{2}/\sin\theta_0$ with surface tension

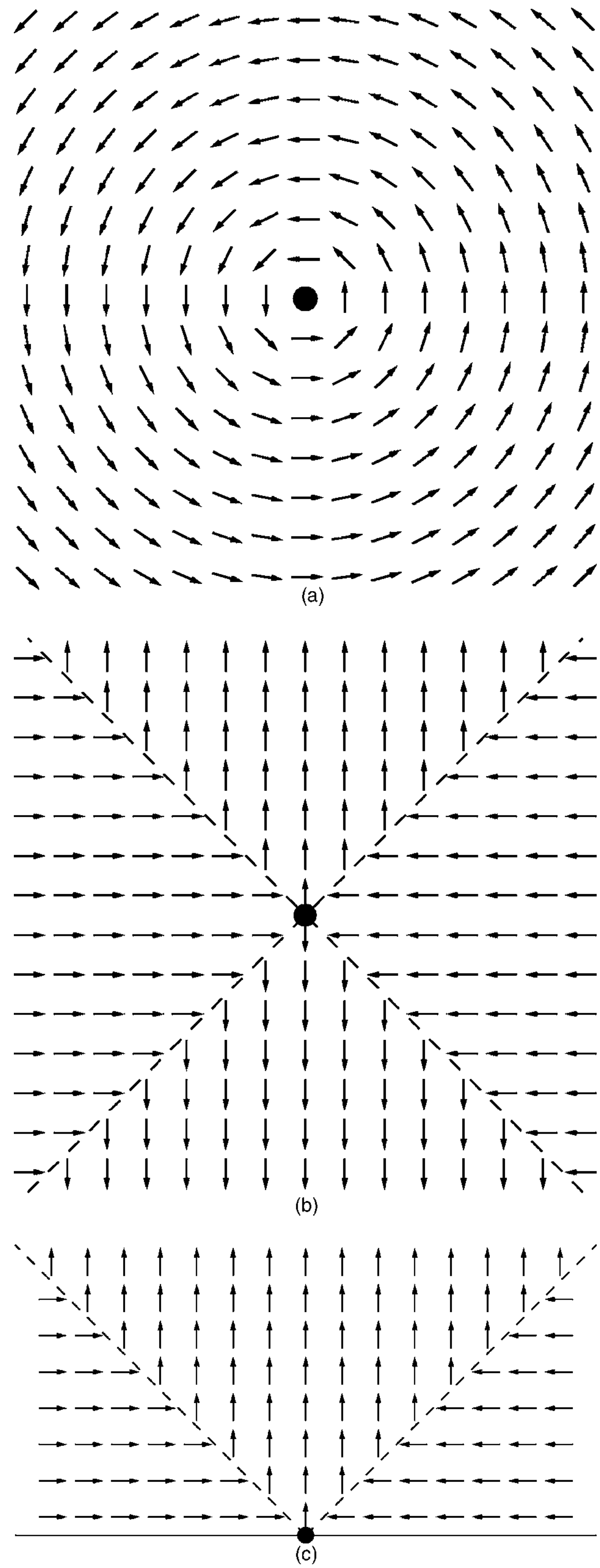

FIG. 1. Top to bottom: a vortex, an antivortex, and a $-1/2$ edge defect in the magnetostatic limit.

$$\sigma = 2\sqrt{2}(\sin\theta_0 - \theta_0\cos\theta_0)A/\lambda. \tag{5}$$

In thinner films ($t \lesssim \lambda$) the magnetostatic term is nonlocal and the Néel walls acquire long tails.[3]

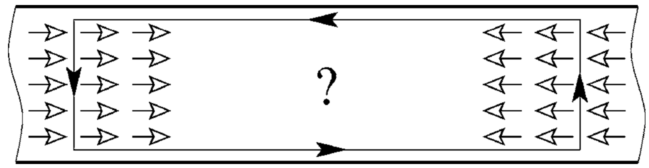


FIG. 2. Determination of the topological charges at the edges and in the bulk.

An edge defect[6] with the winding number of $-1/2$ can be constructed by placing the core of a cross tie at the edge of the film, so that the magnetization along the edge is parallel to the boundary (Fig. 1). As the core is circumvented counterclockwise the magnetization rotates clockwise through $\pi$, in agreement with the definition.[6] The energy of such a defect is also given by Eq. (3).

We have not been able to find any configuration that would contain a $+1/2$ edge defect and be free from magnetic charges. It looks likely that the $+1/2$ defects carry a finite amount of magnetic charge and thus have a substantially higher magnetostatic energy than the other three types of defects described above. This may indicate that, in the limit where the magnetostatic energy dominates, a $+1/2$ defect will decay into a vortex ($n=+1$) and an edge defect ($n=-1/2$). The $+1/2$ defects are stable in the exchange limit.[6,7]

The defects discussed in this paper determine the properties of domain walls in nanomagnetic strips. Postponing a detailed discussion to the accompanying paper[8] here we make two general observations that place important constraints on the possible composition of a domain wall.

First, a domain wall in a strip must contain (at least) one edge defect at each edge. This follows from the definition of their winding numbers.[6] Moving along the upper/lower edge (Fig. 2) one finds that the magnetization rotates through the angle $-2\pi n_{1,2}$. In the presence of a domain wall, the edge winding numbers $n_1$ and $n_2$ are half integers.

Second, the total topological charge of a domain wall, including the winding numbers of the edges and the bulk, must be zero. This can be seen by drawing a contour enclosing the domain wall (Fig. 2) and noting that the total angle of rotation along that contour $-2\pi n_1 - 2\pi n_2$ also equals $2\pi n$, where $n$ is the winding number in the bulk. Hence $n+n_1+n_2=0$.

Thus domain walls with the smallest number of defects may contain (a) two edge defects with winding numbers of $+1/2$ and $-1/2$ and no bulk defects, (b) two $+1/2$ edge defects and one antivortex, and (c) two $-1/2$ edge defects and one vortex. Case (a) corresponds to the transverse wall, which is indeed the lowest-energy domain wall in the exchange limit.[6] In the opposite magnetostatic limit one must minimize the number of $+1/2$ edge defects (which have a high magnetostatic energy). Therefore it is reasonable to expect that the lowest-energy domain walls in this limit are of type (c). Both experimental observations[4] and numerical simulations[5] are consistent with this proposition. See the companion paper[8] for details.

Much of the recent experimental effort in nanomagnetism has been devoted to the study of vortices.[11,15,16] Given an equal (if not greater) importance of edge defects in determining the properties of domain walls, a careful examination of topological defects at the edge is highly desired.

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[1] J.-G. Zhu, Y. Zheng, and G. A. Prinz, J. Appl. Phys. **87**, 6668 (2000).
[2] F. Q. Zhu, D. Fan, X. Zhu, J.-G. Zhu, R. C. Cammarata, and C.-L. Chien, Adv. Mater. (Weinheim, Ger.) **16**, 2155 (2004).
[3] A. Hubert and R. Schaefer, *Magnetic Domains* (Springer, Berlin, 1998).
[4] M. Kläui *et al.*, Phys. Rev. B **68**, 134426 (2003).
[5] R. D. McMichael and M. J. Donahue, IEEE Trans. Magn. **33**, 4167 (1997).
[6] O. Tchernyshyov and G.-W. Chern, Phys. Rev. Lett. **95**, 197204 (2005).
[7] M. Kurzke, Calculus Var. Partial Differ. Equ. (to be published); Max Planck Institute for Mathematics in the Sciences, Report No. 14/2004 (unpublished).
[8] H. Youk, G.-W. Chern, K. Merit, B. Oppenheimer, and O. Tchernyshyov, J. Appl. Phys. (these proceedings); e-print cond-mat/0508741.
[9] R. V. Kohn and V. V. Slastikov, Proc. R. Soc. London, Ser. A **461**, 143 (2004).
[10] A. DeSimone, R. V. Kohn, S. Müller, and F. Otto, Max Planck Institute for Mathematics in the Sciences, Report No. 80/2004 (unpublished).
[11] A. Wachowiak, J. Wiebe, M. Bode, O. Pietzsch, M. Morgenstern, and R. Wiesendanger, Science **298**, 577 (2002).
[12] H. A. M. van den Berg, J. Appl. Phys. **60**, 1104 (1986).
[13] P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, Cambridge, 2000).
[14] M. Löhndorf, A. Wadas, H. A. M. van den Berg, and R. Wiesendanger, Appl. Phys. Lett. **68**, 3635 (1996).
[15] T. Shinjo, T. Okuno, R. Hassdorf, K. Shigeto, and T. Ono, Science **289**, 930 (2000).
[16] S.-B. Choe, Y. Acremann, A. Scholl, A. Bauer, A. Doran, J. Stöhr, and H. A. Padmore, Science **304**, 420 (2004).