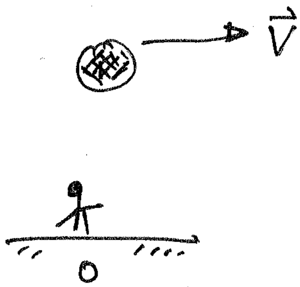
Special theory of relativity

[Mon.] July 21, 08

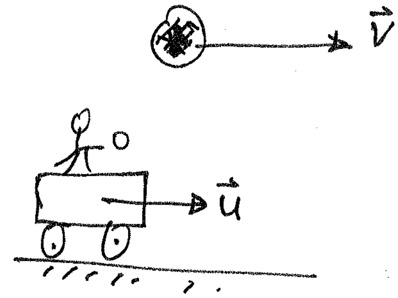
- velocity of an object is always measured with respect to another object (called the "reference frame").

e.g.



"O" observer (stationary on ground.)

["O" sees the particle moving w/ constant velocity $\vec{V}$]



"O" observer moving with constant velocity $\vec{u}$

["O" sees the particle moving w/ constant velocity $\vec{V}-\vec{u}$.]

- what about light?

  * Light (EM wave) is special in that it requires no medium for propagation. E.g. Sound waves require atoms (in the air say.), wave on a string requires the atoms making up the string, heat wave requires particles to transfer thermal energy so, etc..

But light (EM waves) requires none of that!

Recall that last week, we derived the EM wave eqns in free space (vacuum.):

$$\left.\begin{aligned} \frac{\partial^2 \vec{E}}{\partial t^2} &= c^2 \nabla^2 \vec{E} \\ \frac{\partial^2 \vec{B}}{\partial t^2} &= c^2 \nabla^2 \vec{B}. \end{aligned}\right\}$$

$\boxed{c \equiv \text{speed of light in vacuum.}}$

* What is "c" in that equation measured with respect to? Does light appear to be stationary if you happen to travel at the speed of light?

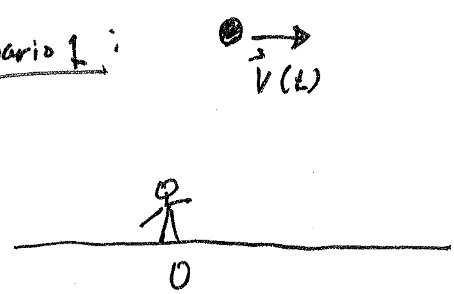
* At the turn of the 20th century, one main hypothesis was that "c" is measured with respect to a material called "ether" which pervades entire universe.

(Search & read about "Michaelson-Morley" experiment.)
↳ Failed to detect existence of ether.

What's going on? How do we answer this question?

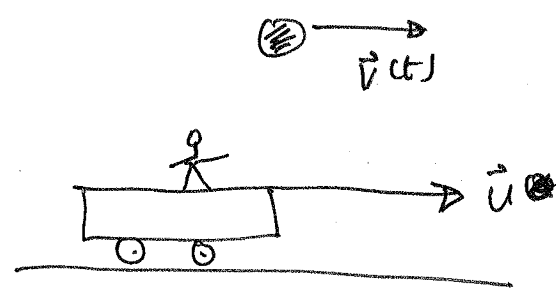
Consider the following : $\vec{F} = \dfrac{d(m\vec{v})}{dt}$. ← Newton's 2nd law.

Scenario 1 :



Stationary observer watches a particle moving with some variable velocity $\vec{v}$.

Scenario 2 :



Observer watches the same particle but is now moving with a constant velocity $\vec{u}$ with respect to the ground.

In scenario 1 : observer would describe the particle's motion using:

$$\vec{F}_1 = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt}$$

In scenario 2 : observer would describe the particle's motion using:

$$\vec{F}_2 = \frac{d(m(\vec{v}(t) - \vec{u}))}{dt}$$

← since observer sees that the particle is moving with velocity $\vec{v}(t) - \vec{u}$ relative to him / her.

$$= m\frac{d\vec{v}}{dt} - \underbrace{m\frac{d\vec{u}}{dt}}_{0}$$

(since $\vec{u}$ constant velocity)

$$= m\frac{d\vec{v}}{dt}$$
$$= \vec{F}_1 \implies \boxed{\vec{F}_1 = \vec{F}_2}$$

So both observers would be describing the same physics.

~~Also~~ or, The actual velocity of particle observed by the 2 observers is different in the 2 scenarios, but that's not what I mean by both of them describing the same physics. What I mean is that the motion of particle can still be described by Newton's 2nd law: $\boxed{\vec{F} = \frac{d(m\vec{v})}{dt}}$

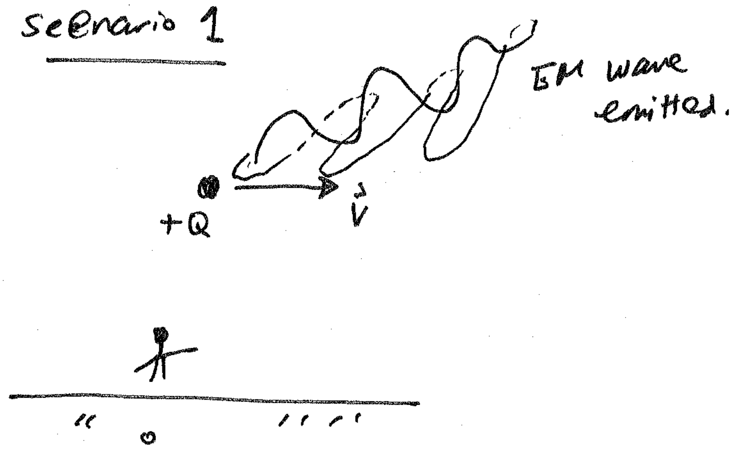
where $\vec{v}$ is observed velocity of particle, which depends on observer's frame of reference.

Can the same thing be said about Electromagnetic phenomena? That is, ~~do Maxwell's equations~~ can you just apply Maxwell's eq'ns by modifying velocity of charged particle $\vec{V}$ by $\vec{V}-\vec{u}$?

Ans: No! It turns out that Maxwell's eq'ns predict non-sense if we try the same technique of adjusting the velocity of $\vec{V}$ by $\vec{V}-\vec{u}$. What the theory predicts doesn't match electromagnetic phenomena actually observed.

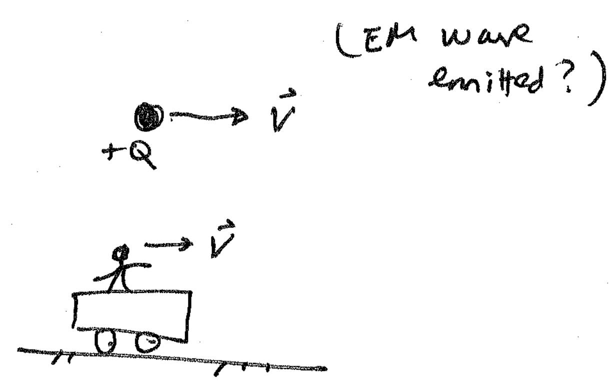
Physically, we can see why this must be so:

Scenario 1



Observer at rest with respect to ground.
Positive charge +Q moves with velocity $\vec{V}$ with respect to ground.

Scenario 2



Both observer and positive charge +Q moves with velocity $\vec{V}$ with respect to ground.

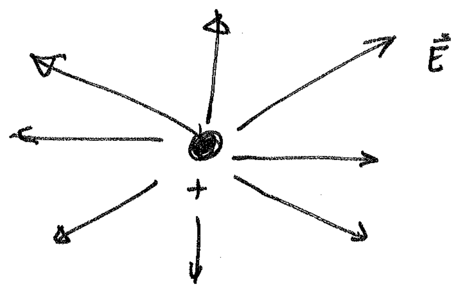
• What does the observer see in each scenario?

In scenario 1: observer sees electromagnetic wave emitted by the charge.

($\vec{E}$ and $\vec{B}$ Both exist)
(Dynamic field (varying over time.)

In scenario 2: observer sees the +Q charge at "rest" relative to him/her.

So, the observer should just see:



- Radially emitted $\vec{E}$. (static field) (Not varying over time.)
- No magnetic field anywhere: $\vec{B} = 0$.

Can we just adjust the velocity of charged particle and get this?
(from $\vec{v}$ to $\vec{v} - \vec{v} = 0$)

Ans: No.

To see this, notice that in scenario 1, the EM wave emitted would look like ~~Electromagnetic wave~~

$$\vec{E}(z,t) \sim \sin(kz - \omega t)$$

$$\vec{B}(z,t) \sim \sin(kz - \omega t)$$

Looking at these eq'ns, How would $\vec{B}$ disappear while $\vec{E}$ does not, by adjusting $\vec{v} \to \vec{v} = 0$?

Ans: We can't.

Solution to the whole dilemma : In 1905, ~~the discrep~~ Einstein pointed out that Maxwell's eq'ns are correct, and that there's no such thing as ether. More importantly, he pointed out the shortage in classical physics, namely, $\cancel{\vec{V} \rightarrow \vec{V} - \vec{U}}$.

only an approximation when the speed of observer $|\vec{U}|$ is much less than speed of light c.

And as for speed of light, it's a true constant in that no matter what speed you are traveling at, as an observer, you always see light traveling at speed c relative to you.

Einstein's special theory of relativity : (2 postulates forming the basis of special relativity.)

1. Principle of relativity : The laws of physics apply in all inertial reference systems

   (Inertial reference system is one in which observer is moving at constant velocity.)

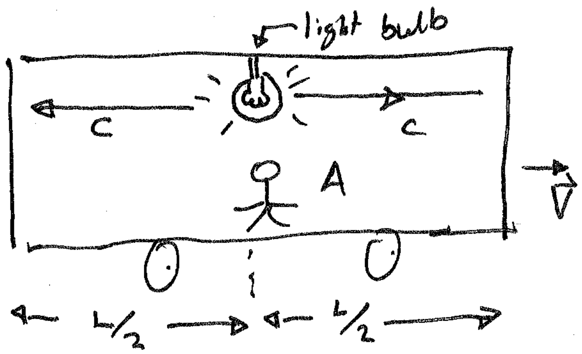
2. Universal speed of light : speed of light in vacuum is the same for all inertial observers, regardless of the motion of the source.

With special relativity in place, we let go of our everyday "common sense", which ~~is ingrained~~ is based on our every day experiences dealing with speeds much less than speed of light in vacuum (3×10^8 m/s.).

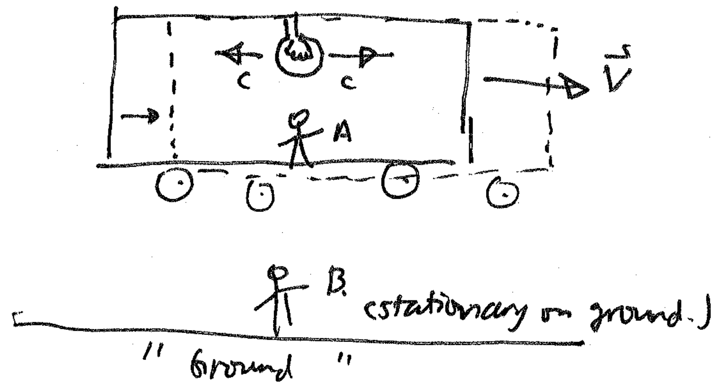
1.) Two events that are simultaneous in one inertial system are not, in general, simultaneous in another.:

To see this:

Scenario 1:



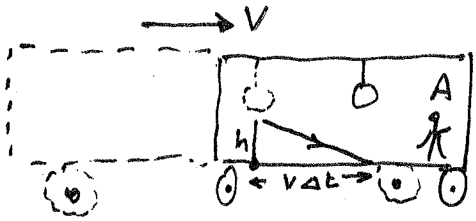
Scenario 2:

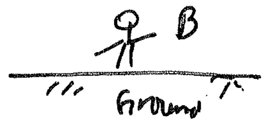


In scenario 1, the person A standing inside the train moving at constant velocity $\vec{V}$, is in an inertial reference frame. He/she sees light pulse emitted from the lightbulb hit both ends of train at the same time.

In scenario 2: Person B sees light pulse emitted from the bulb (one to the left, the other moving to the right.) He/she sees the pulse moving to the left hit the left end of train (rear-end) before the pulse moving to the right hits the front-end (right end) of train.

2.) Time dilation

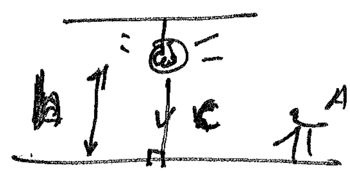




Observing a train moving at constant speed V to the right (relative to the ground), as an observer on the ground,

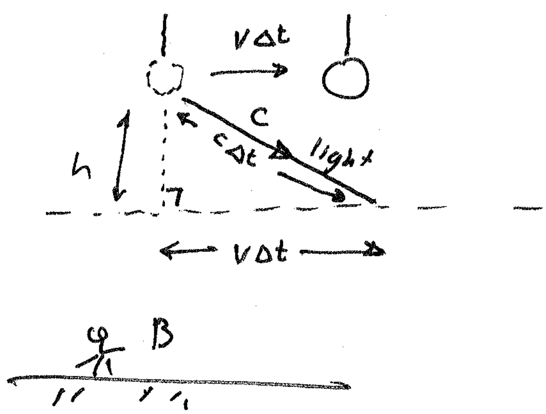
Imagine a light ray leaving the lightbulb in the train, hitting the pt. on the floor of train directly below it.

As an observer A inside the train, he / she sees the light travel vertically down a distance: h



To him/her,
this takes time $\frac{h}{c} = \bar{\Delta t}$.

But as an observer on the ground: he / she Observes:



$$\sqrt{h^2 + (v\,\Delta t)^2} = c\,\Delta t$$

$$\Rightarrow \boxed{\Delta t = \frac{h}{c}\,\frac{1}{\sqrt{1-(V/c)^2}}}$$

~~Observer B sees~~
To observer B on the ground, it takes Δt amount of time for the same pulse of light to hit the ground directly below the light bulb.

Notice that $\Delta t \neq \bar{\Delta t}$
if $V \neq 0$.

Therefore: $\boxed{\Delta\bar{t} = \sqrt{1-(V/c)^2}\,\Delta t}$

And we commonly write, by convention, $\boxed{\gamma = \dfrac{1}{\sqrt{1-(V/c)^2}}.}$

so: $\boxed{\Delta\bar{t} = \dfrac{\Delta t}{\gamma}}$ (notice that $\gamma \geq 1$)

This phenomena ($\Delta t \neq \Delta\bar{t}$ if $V \neq 0$) is called time dilation.

Conclusion: Moving clocks run slow. (i.e. $\Delta\bar{t} < \Delta t$.)

Paradox #1: "Twin" paradox: On her 21st birthday, an astronaut takes off in a rocket ship at a speed of $\frac{12}{13}c$.

After 5 yrs have elapsed on her watch, she turns around and heads back at the same speed to rejoin her twin brother, who stayed at home.

Qu: How old is each twin at their reunion?

- The traveling twin: Aged total of 10 yrs.

(→ 5 years on the way to.
← Another 5 yrs on the way back)

⇒ She arrives back home to celebrate her 31st birthday.

- What about the twin that stayed back at home?

Ans: As viewed from Earth, the moving clock has been running slow by a factor: $\gamma = \dfrac{1}{\sqrt{1-(12/13)^2}} = 13/5$.

Thus, time elapsed on Earth is $\frac{13}{5} \times 10 = 26$.

$\Rightarrow$ Her ~~Earth~~ brother will be 47 yrs old by the time she comes back to Earth.

$\Rightarrow$ He is now 16 yrs older than her twin sister!

<u>Caution</u>: The twin sister will <u>not</u> have lived longer than her brother, she's just lived "slower".

During her space trip, <u>all</u> of her biological processes such as metabolism, pulse, thought, and speech etc. have <u>all</u> been time dilated just like her watch.

~~Back to length contraction~~

But <u>Twin paradox arises</u>: Relative to the traveling sister, it's her twin <u>brother</u> on Earth who's been speeding away from her at speed of $\frac{12}{13}c$.

So reversing the argument above, shouldn't her brother have aged less than her??

<u>Ans</u>: <u>No!</u> The reason is that the sister has actually been in 2 <u>different</u> inertial frames in the duration of her round trip.

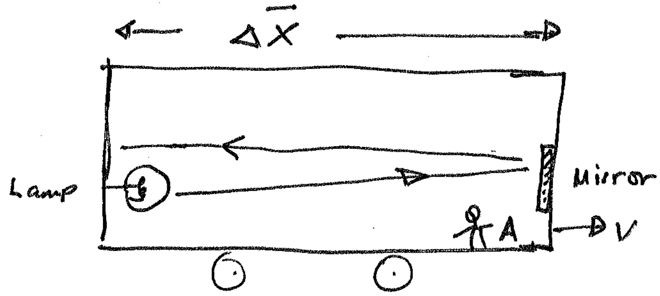
(one inertial frame on the way <u>to</u>
another inertial frame on the way <u>back</u>.)

Between these 2, she accelerated to turn back.

3) Length (Lorentz) contraction:

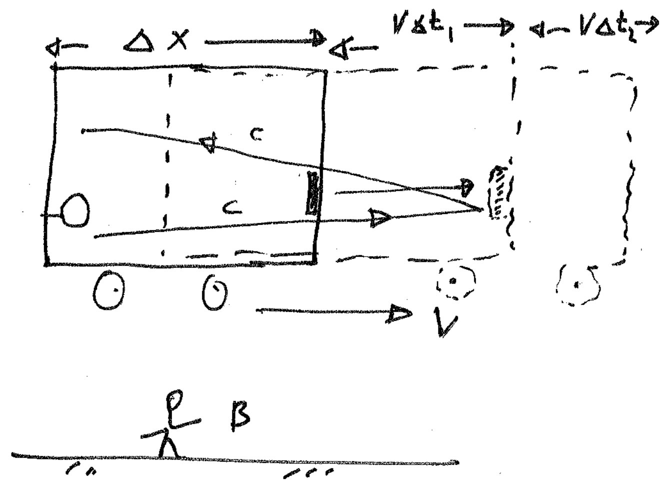
light bounces from mirror at the front end of train, and hits the rear end of train

Scenario 1



Observer A on the moving train.

Scenario 2



Ground.

Note: Train moves with constant speed V to the right, relative to the ground.

Scenario 1: According to observer A on the train:

The length of train is $\Delta\bar{x}$, as measured by A on the train.

So, the total time for the light pulse's round trip (back → front mirror → back) is

$$\boxed{\Delta\bar{t} = \frac{2\,\Delta\bar{x}}{c}}$$

Scenario 2: According to observer B on the ~~train~~ ground.

$\Delta x \equiv$ length of moving train as measured by observer B.

Δt_1 = time taken for light ray's motion (rear end → front end mirror):

$$= \frac{\Delta x + V\Delta t_1}{c}$$

Δt_2 = time taken for light ray's motion (front end mirror → rear end)

$$= \frac{\Delta x - V\Delta t_2}{c}.$$

So,

$$\Delta t_1 = \frac{\Delta x}{c-v} \quad , \quad \Delta t_2 = \frac{\Delta x}{c+v}$$

$$\therefore \quad \Delta t_{total} = \Delta t_1 + \Delta t_2 = 2\frac{\Delta x}{c}\frac{1}{(1-v^2/c^2)}$$

But, from time dilation:
$$\Delta \bar{t} = \frac{\Delta t}{\gamma} = \sqrt{1-(v/c)^2}\,\Delta t.$$

$$\therefore \quad \boxed{\Delta \bar{x} = \gamma \Delta x = \frac{1}{\sqrt{1-(v/c)^2}}\Delta x}$$

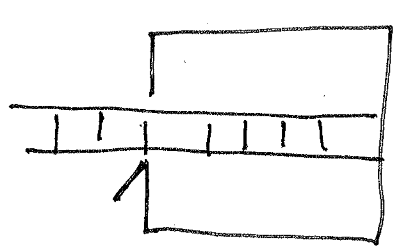
Conclusion: Moving objects are shortened.

(Notice that the same factor γ is involved)

Paradox #2 : Barn and ladder paradox

- No direct experimental confirmation of Lorentz contraction exists. (But there is one for time dilation).

Scenario 1

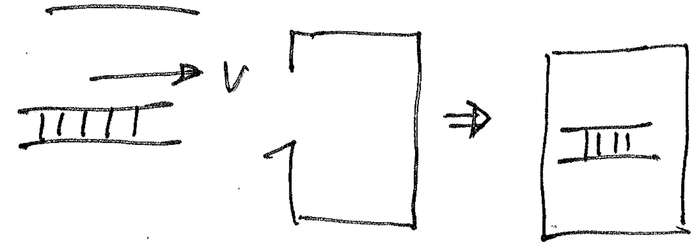


Barn

Ladder doesn't fit into the barn.

Ladder is too long.

Scenario 2



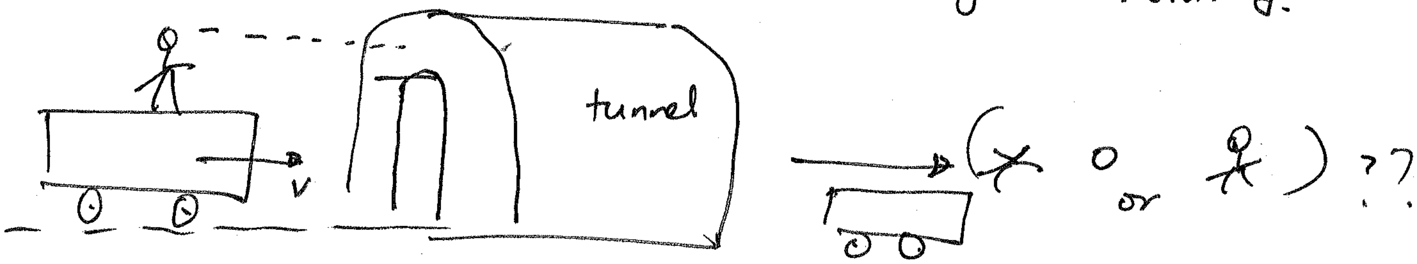
Fast moving ladder. ⇒ Length shortened so it should fit into the barn.

Paradox?? Ans No!

Note: Dimensions perpendicular to the velocity are not contracted.

(Argument of Taylor & John. A. Wheeler)

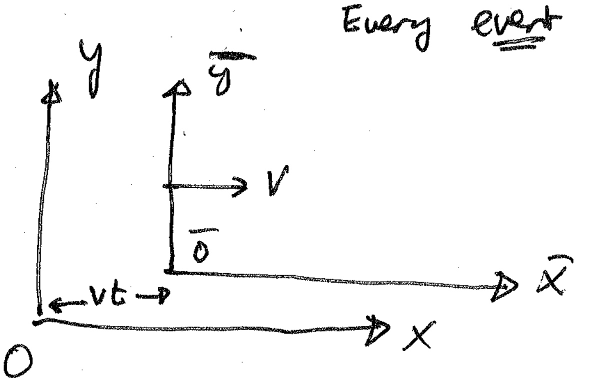
↳ Expert on general relativity.



Will the person's head be chopped off by the time the train emerges?

Putting everything together: Lorentz transformations

Every event can be described by (x, y, z, t) by O
$(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ by $\bar{O}$.



$$\cancel{\left.\begin{aligned} \bar{x} &= x - vt \\ \bar{y} &= y \\ \bar{z} &= z \\ \bar{t} &= t \end{aligned}\right\}}$$

← classical physics says this. Wrong!!

(O & $\bar{O}$ are 2 inertial reference frames.

$\bar{O}$ is moving at speed V relative to O, to the right.

Due to Lorentz & Einstein:

Lorentz transformation →

$$\boxed{\begin{aligned} \bar{x} &= \gamma(x - vt) \\ \bar{y} &= y \\ \bar{z} &= z \\ \bar{t} &= \gamma\left(t - \frac{v}{c^2}x\right) \end{aligned}}$$

Though difficult to derive given our time and background: L11-14

EM fields transform according to Lorentz transformation as well.