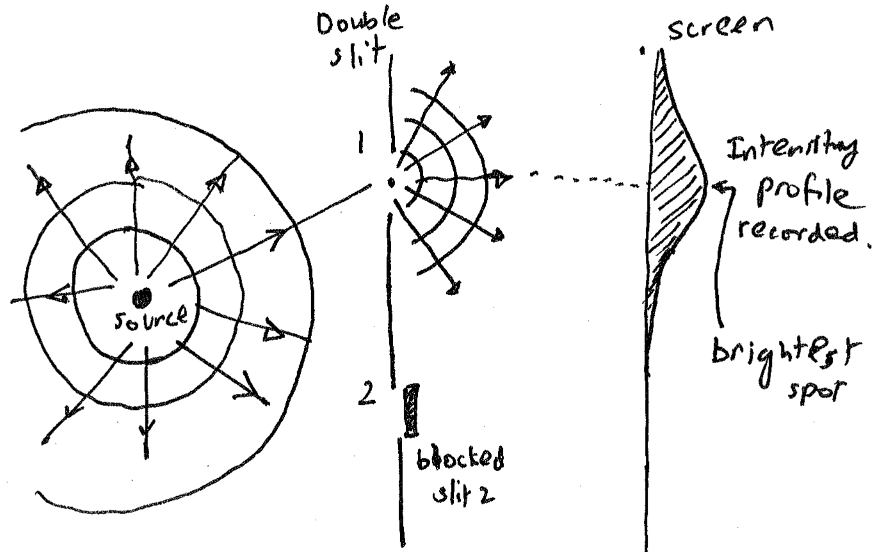
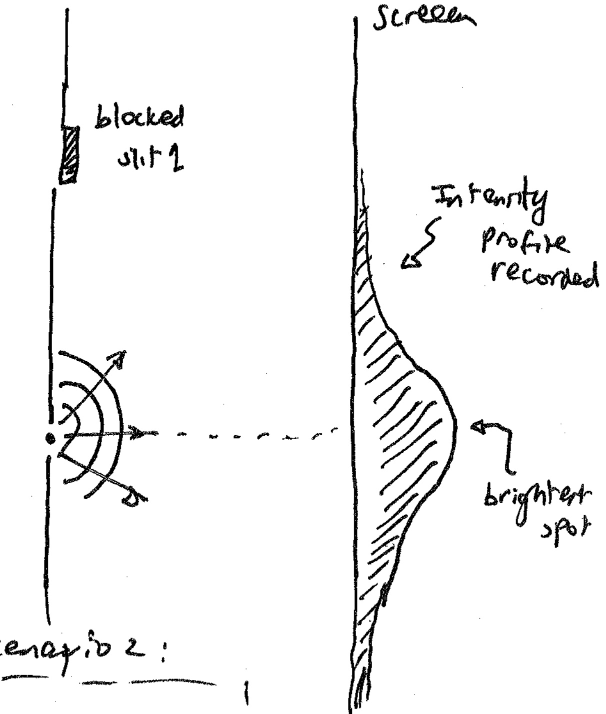
Survey of key ideas in Quantum Mechanics

Young's Double slit expt.

[wed.] July 23, 08

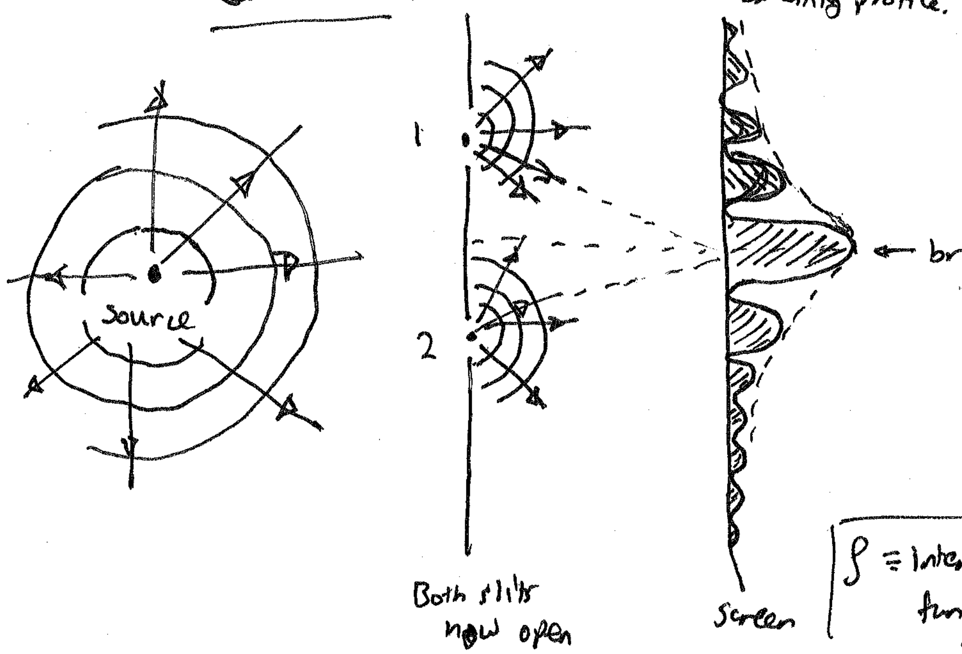


scenario 1:



scenario 2:

scenario 3:



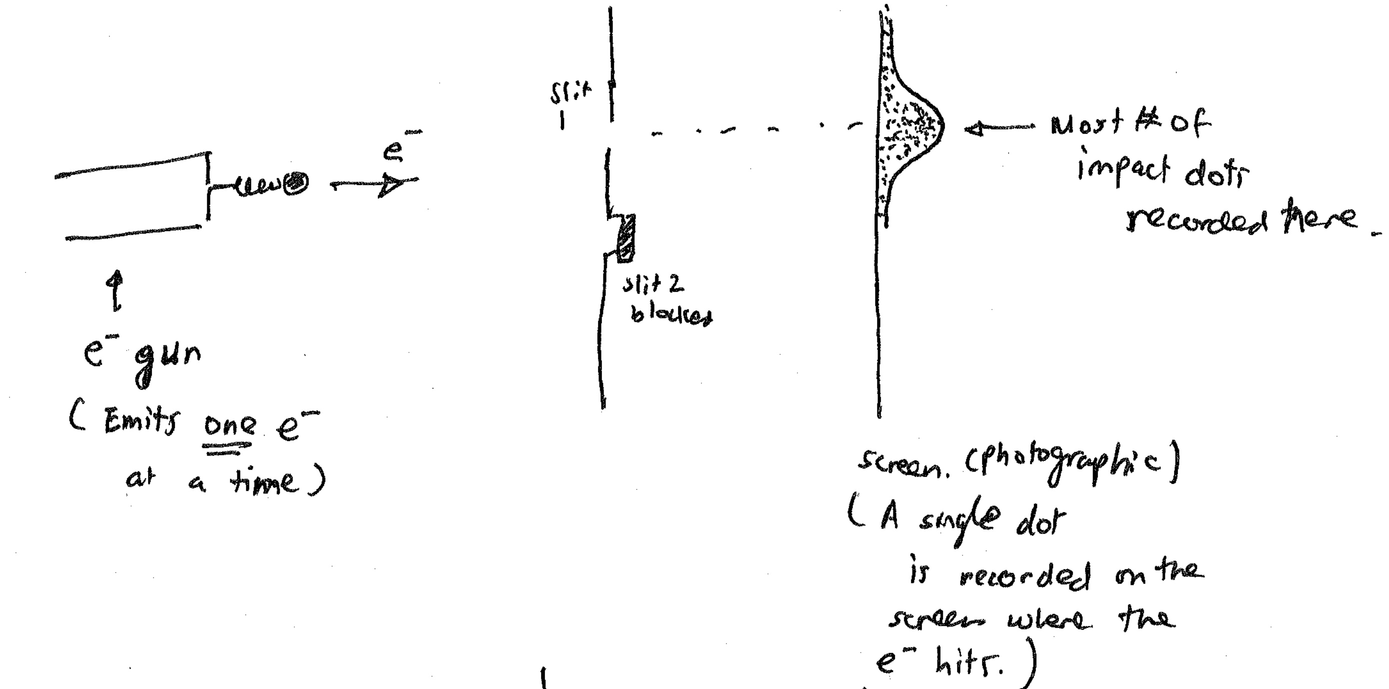
← Result of constructive & destructive interferences between light diffracted from the 2 slits.

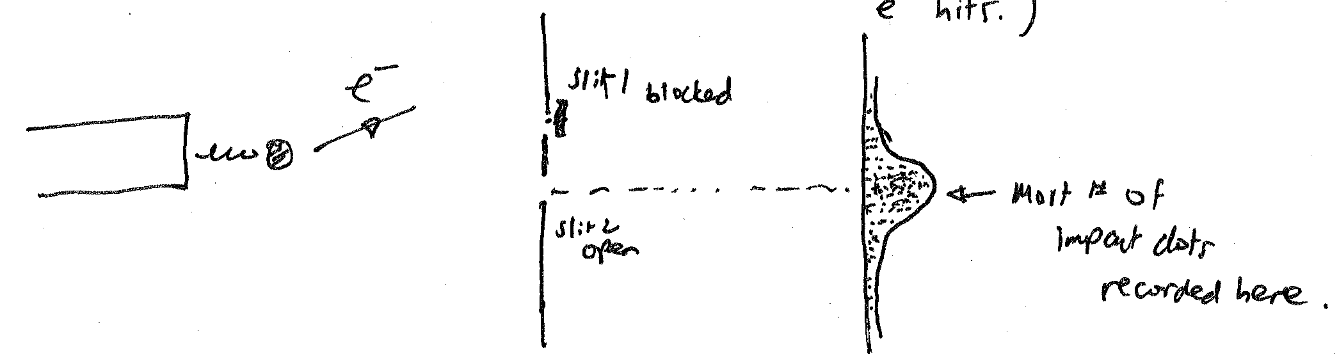
ρ ≡ Intensity function on screen.

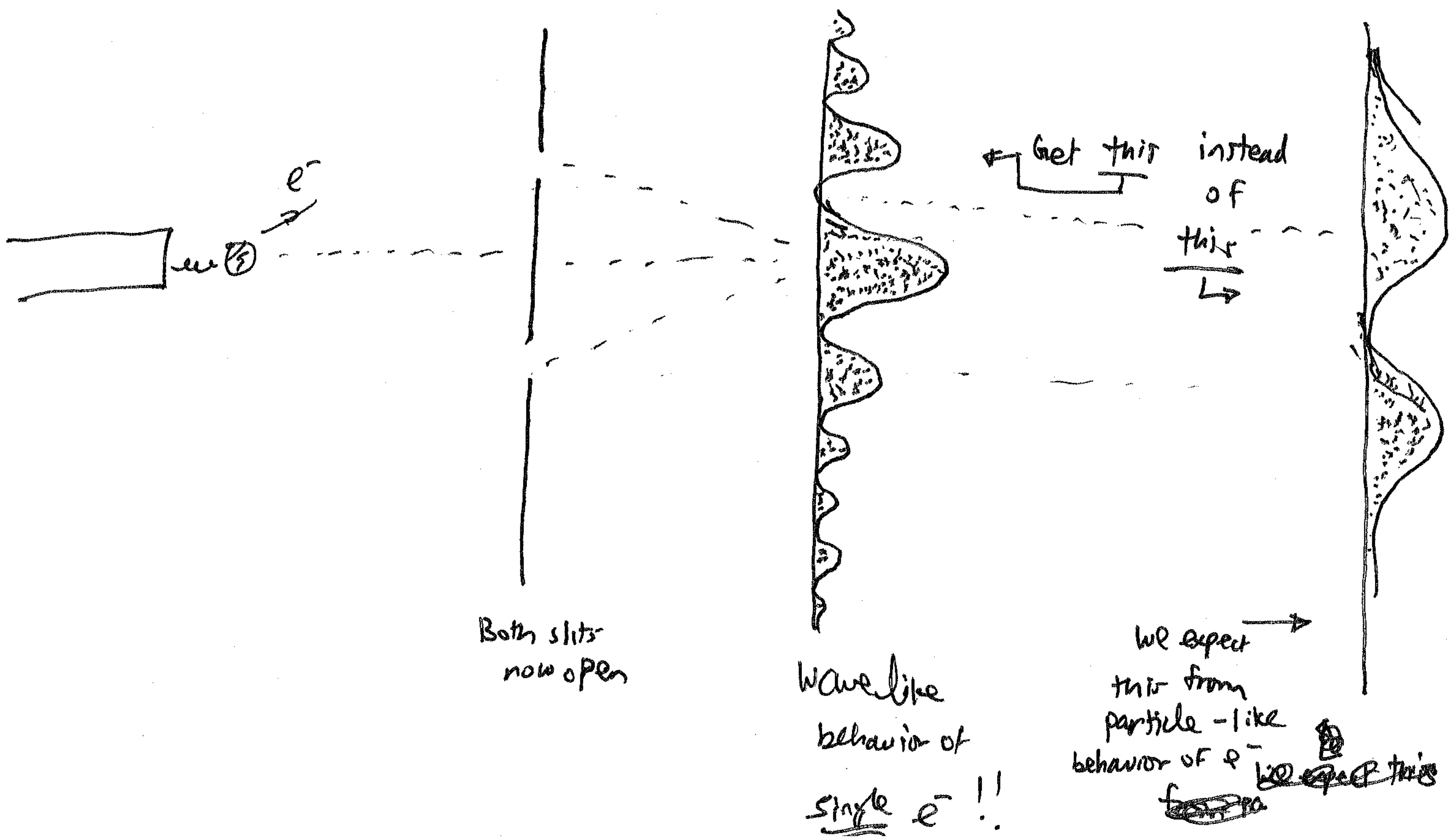
Wave nature of light gives this pattern.

- source ≡ light source.
- Can also do this experiment with ripples on surface of water.

But next, a similar experiment with an electron.

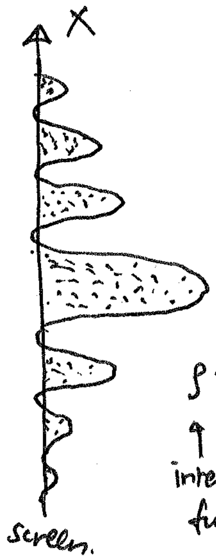






Is the e^- interfering with itself ?!?!

Ans : Probabilistic interpretation :



$\rho = |\psi|^2$. (i.e. $\rho(x) = |\psi(x)|^2$.)

intensity function on screen.

* An interference pattern and its probability interpretation: Each e^- makes a localized impact on the screen. The interference pattern becomes visible after the impact of many e^-'s with the same "wave function" $\psi(x, t)$

ψ ← Greek letter "Psi"

If we accept that the wave function $\psi(\vec{x}, t)$ gives probability distribution $\rho(\vec{x}, t) = |\psi(x, t)|^2$ that an e^- occupies position x,

then "electron wave" $\psi_1(x, t)$ and "electron wave" $\psi_2(x, t)$,

which cause screen to darken according to $\rho_1(x) = |\psi_1(x, t)|^2$ and

$\rho_2(x) = |\psi_2(x)|^2$, respectively,

then when both slits are open: $\psi_1(x, t) + \psi_2(x, t)$ = resultant e^--wave from interference of both e^- waves from both slits.

and

$$\boxed{\rho(x, t) = |\psi_1(x, t) + \psi_2(x, t)|^2}$$

Notice that $\rho(x, t) = |\psi(x, t)|^2$ is the probability density (over space)

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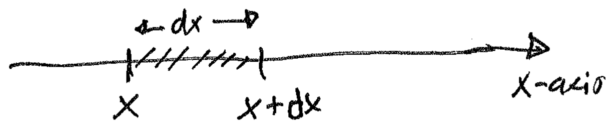
And $\rho(x,t)\,dx = |\psi(x,t)|^2 dx$ is the probability that the Particle (e^- in this case) hits the screen within the interval $(x, x+dx)$.

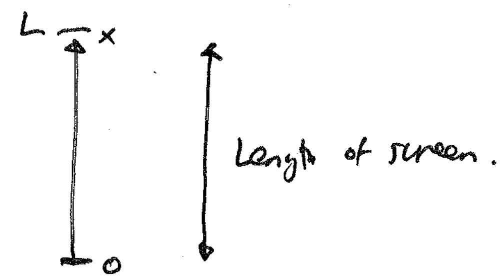


[i.e. e^- impacts position on the screen between x and $x+dx$]

Now, since e^- hits some place on the screen (i.e. it doesn't disappear), the probability that e^- hits some place on screen is 1.

$$\Rightarrow \quad 1 = \int_0^L \rho(x,t)\,dx$$

$$= \int_0^L |\psi(x,t)|^2 dx$$



And if screen stretches from $-\infty$ to $+\infty$:

$$\boxed{1 = \int_{-\infty}^{+\infty} \rho(x,t)\,dx = \int_{-\infty}^{\infty} |\psi(x,t)|^2 dx}$$

What eqn describes "Probability wave" $\psi(x,t)$?

says "$V(x)\psi(x,t)$" (sorry about messy writing)

Ans: Schrödinger eqn

$$\boxed{i\hbar \frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t)}$$

$V(x)$ = potential energy

$\hbar$ = planck's constant.

m = mass of particle

$[\hbar]$ = Energy × time (J · s)

↑ Basically, "All" of quantum mechanics. (plus some postulates)

Notice that if $\psi(x,t) = Ae^{i(kx-\omega t)}$ ← $\mathbb{C}$-number representation of plane wave

and $V(x) = 0$

(Free particle : No potential energy).
So E_{total} = Just kinetic energy of particle.)

Then does our guess $\psi(x,t)$ satisfy the Schrödinger eqn ?

Check : LHS : $i\hbar \frac{\partial \psi}{\partial t} = \hbar\omega A e^{i(kx-\omega t)} = \hbar\omega \psi(x,t)$

RHS : $\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = \frac{\hbar^2 k^2}{2m} \psi(x,t)$

In order for
∴ LHS = RHS , we need $\boxed{\frac{\hbar^2 k^2}{2m} = \hbar\omega}$ ← Has units of energy since

$[\hbar\omega] = [\hbar][\omega]$
$= [\text{Energy} \cdot \cancel{\text{time}}] \frac{1}{\cancel{\text{time}}}$
$= \text{Energy}.$

Kinetic energy of particle.

Notice that $[k] = \frac{1}{\text{length}}$

In fact, we know that $\boxed{k = \frac{2\pi}{\lambda}}$

$\lambda \equiv$ wave length of a "particle" !!

(wave-particle duality)

Subtlety : But remember, we are describing the probability wave function with $\psi(x,t)$.

Equally acceptable is $\psi(x,t) = Be^{-ikx - i\omega t}$.

left moving "probability current"

· You can check for yourself that Schrödinger eq'n is linear differential eq'n; $\Rightarrow \psi(x,t) = A e^{i(kx-\omega t)} + B e^{i(-kx-\omega t)}$

$$= \boxed{\left[A e^{ikx} + Be^{-ikx}\right] e^{-i\omega t}}$$

general solution to Schrödinger eq'n when $V(x)=0$.

plane waves $\rightarrow + \leftarrow$

* Probabilistic description gives rise to "uncertainty principle"

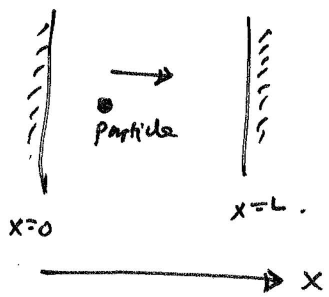
Example : $\boxed{\Delta p \, \Delta x \geq \hbar/2}$

$\Delta p \equiv$ uncertainty (precision) in momentum determination

$\Delta x \equiv$ uncertainty (precision) in position determination.

Boundary conditions: Ex: Consider 2 walls.
and particle confined between the 2 walls.
No potential energy, just kinetic.

Want to solve $i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$. between $0 \leq x \leq L$



We know from previous page: $\psi(x,t) = [Ae^{ikx} + Be^{-ikx}]e^{-i\omega t}$
is a solution to the eqn w/o the boundary conditions.

Imposing the BCs, gives:

$$\begin{cases} \psi(x=0,t)=0 \\ \psi(x=L, t)=0 \end{cases}$$

Exactly the problem we solved (standing waves on a string fixed at 2 walls!)!

$$\Rightarrow \quad \psi_n(x,t) = [A_n e^{ik_n x} + B_n e^{-ik_n x}] e^{-i\omega_n t}$$

$$\boxed{k_n = \frac{n\pi}{L}} \qquad n=1,2,3 \ldots$$

Btw $E = \frac{\hbar^2 k^2}{2m}$ $\Rightarrow$ But k only takes on particular values, so does energy!

$$\Rightarrow E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

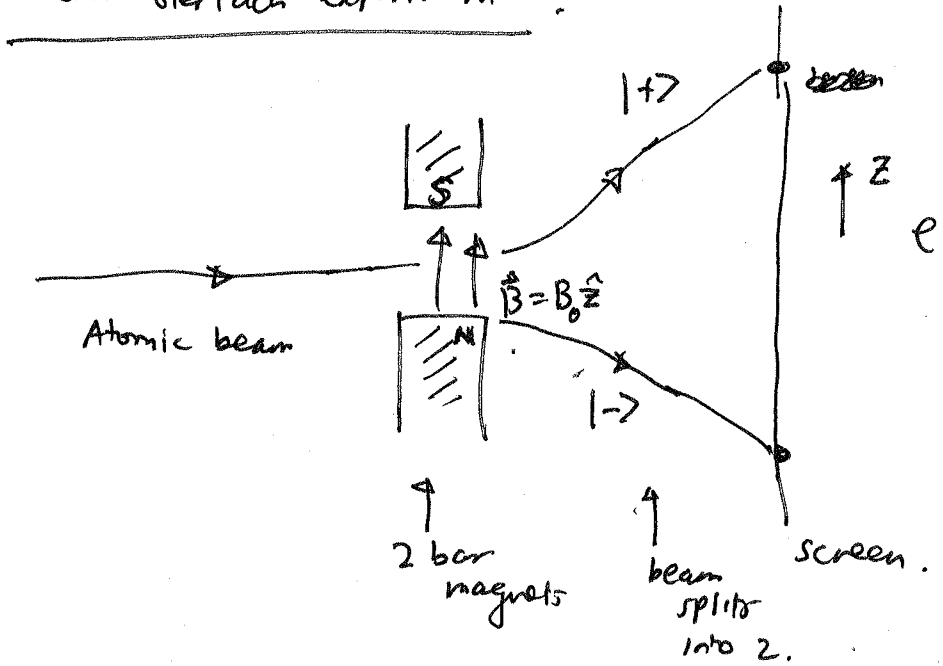
$$E_n = \boxed{n^2 \left(\frac{\hbar^2 \pi^2}{2mL^2}\right)}$$

$n=1,2,3 \ldots$

So, energy can only take on certain values. $\Rightarrow$ $\boxed{\text{Energy is "quantized"}}$

Spin of an e^- : Electron (and any particle) has an intrinsic ~~magnetic~~ angular momentum.

Stern-Gerlach experiment :



e^- has either an "up" spin or a "down" spin.

"up" spin = spin $+1/2$

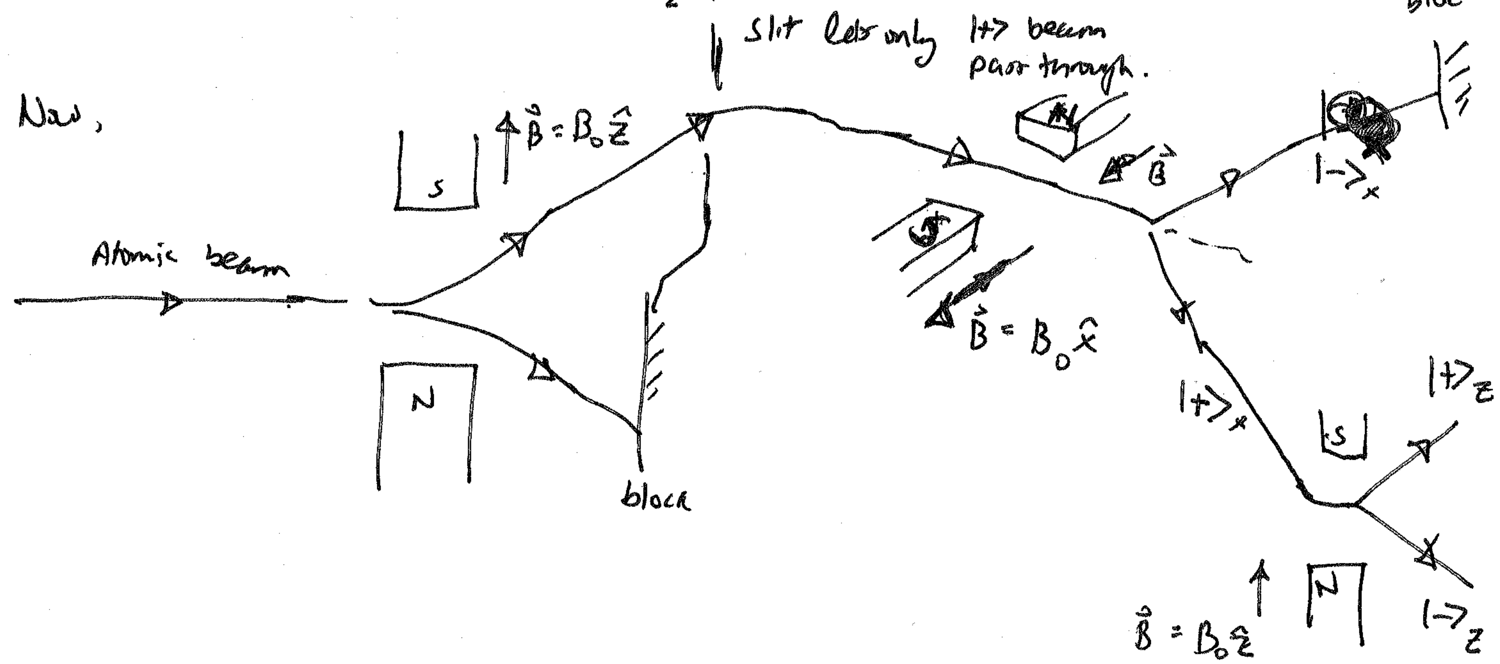
"down" spin = spin $-1/2$

. These 2 are "orthogonal" states.

"up" spin is denoted by "$|+\rangle_z$".

"down" spin is denoted by "$|-\rangle_z$".

Now,



This is bizarre!!