Solution to Problem 1

(a) $\vec{F} = m\vec{a}$ ← Newton's 2^{nd} law

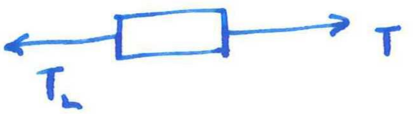
$m = 0 \Rightarrow \vec{F} = 0.$

(b) $\vec{a} = \vec{F}/m$ so if $\vec{F}$ ← total force is not zero, then we have $\frac{\#}{0} = \infty. = \vec{a}$

But we cannot have an infinite acceleration.

So we must have $\vec{F} = 0$ on a massless body.

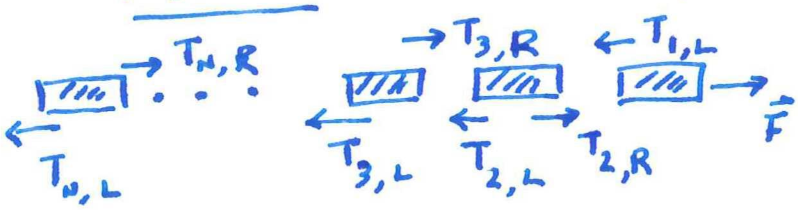
(c)



(no gravity because massless.)

$\vec{F}_{total} = 0 \Rightarrow T_L = T$

(d) <u>method 1</u>: Break string into tiny pieces.



← Break string into N tiny fragments that are joined together.

Let $\vec{T}_{i,L}$ = pull on i^{th} segment from the left.

$\vec{T}_{i,R}$ = pull on i^{th} segment from the right.

The 1^{st} segment touches the person's hand.

so $\vec{T}_{1,R} = \vec{F}$

From part (b); since each segment is massless, we have

$$T_{i,L} = T_{i,R} \qquad T_{i,L} = |\vec{T}_{i,L}|$$

$$(\text{true for all } i) \qquad T_{i,R} = |\vec{T}_{i,R}|$$

And, $T_{2,R} = T_{1,L}$ ← because $T_{1,L}$ (pull on segment 1 from its left) is exerted by segment 2.

so by Newton's 3^{rd} law: $T_{2,R} = T_{1,L}$

And thus $T_{2,R} = T_{1,L} = |\vec{F}|$

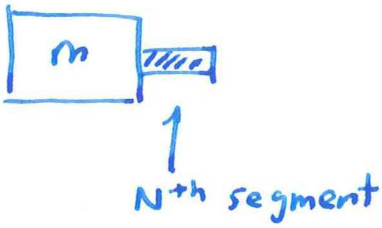
And $T_{2,L} = T_{2,R} \Rightarrow T_{2,L} = |\vec{F}|$.

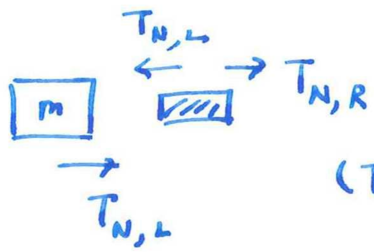
By same reasoning: $T_{3,R} = T_{2,L} \Rightarrow T_{3,R} = |\vec{F}|$

and so on. (in general, we have: $T_{i,L} = T_{i+1,R}$)

Applying this chain of equalities, we get $T_{N,L} = |\vec{F}|$.

And since





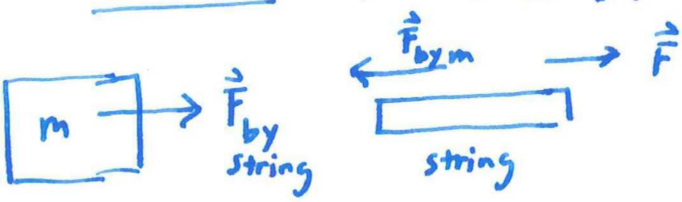
$(T_{N,L} = T_{N,R} = |\vec{F}|)$

($T_{N,L}$ is the pull on m by the N^{th} segment by Newton's 3^{rd} law)

we have

$T_{N,L} = |\vec{F}|$ is the pull on the mass.

<u>Method 2</u>: <u>3 bodies</u>: person, string, block.

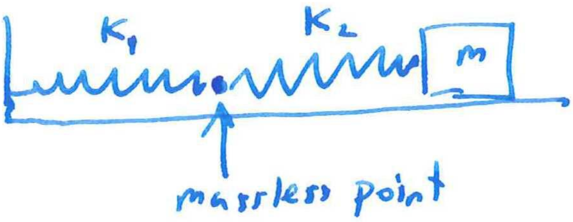


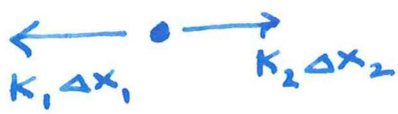
Since string massless, total force on it $= 0$. $\Rightarrow \vec{F} = -\vec{F}_{\text{by } m}$

And by Newton's 3^{rd} law: $\vec{F}_{\text{by string}} = -\vec{F}_{\text{by } m} = \vec{F}$

So indeed the string pulls on m with force $\vec{F}$.

(e)





Must have

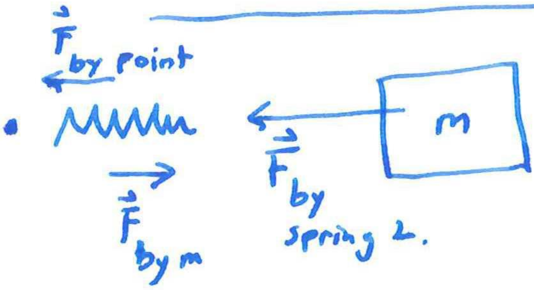
$K_1 \Delta x_1 = K_2 \Delta x_2$

(because massless point)

so net force on it $= 0$.

(f) $k_1 \Delta x_1 = k_2 \Delta x_2 \Rightarrow \boxed{\Delta x_2 = \frac{k_1}{k_2} \Delta x_1}$

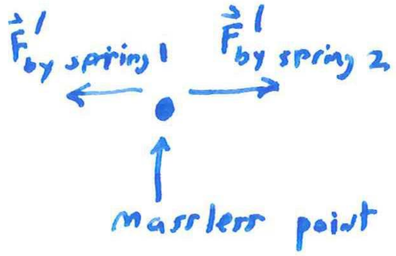
(g) The total force on m:



Here, $\vec{F}_{by\ m}$ = force that m exerts on spring 2.

$\vec{F}_{by\ point}$ = force that the massless point exerts on spring 2.

And since



(where ~~$|\vec{F}_{by}$~~

$|\vec{F}'_{by\ spring\ 1}| = k_1 \Delta x_1$

$|\vec{F}'_{by\ spring\ 2}| = k_2 \Delta x_2$

By Newton's 3rd law: $|\vec{F}'_{by\ spring\ 2}| = |\vec{F}_{by\ point}|$

And spring massless $\Rightarrow |\vec{F}_{by\ point}| = |\vec{F}_{by\ m}|$

And Newton 3rd law $\Rightarrow |\vec{F}_{by\ spring\ 2}| = |\vec{F}_{by\ m}|$

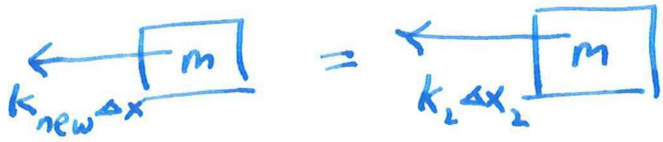
so: $|\vec{F}'_{by\ spring\ 2}| = |\vec{F}_{by\ spring\ 2}| = k_2 \Delta x_2$.

So the block is pulled to left by force $k_2 \Delta x_2$.

(h) $\Delta x = \Delta x_1 + \Delta x_2$

$= \Delta x_1 + \frac{k_1}{k_2} \Delta x_1 \Rightarrow \boxed{\Delta x = \Delta x_1 \left(\frac{k_1 + k_2}{k_2} \right)}$

(i) $k_{new} \Delta x = k_2 \Delta x_2$



$\Rightarrow k_{new} = k_2 \frac{\Delta x_2}{\Delta x}$

$= k_2 \frac{k_1}{\cancel{k_2}} \cancel{\Delta x_1} \cdot \frac{1}{\cancel{\Delta x_1}} \frac{\cancel{k_2}}{k_1 + k_2} \Rightarrow \boxed{k_{new} = \frac{k_1 k_2}{k_1 + k_2}}$

j) 2 identical springs, each with spring constant k_{half}.

so joining them together gives one spring with spring constant k.

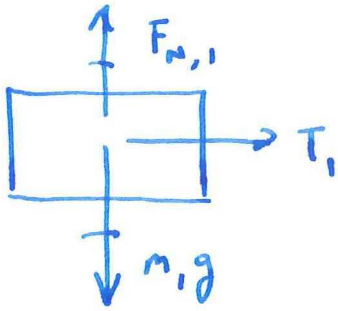
$$\Rightarrow \quad k = \frac{k_{half}^2}{k_{half} + k_{half}} \Rightarrow k = \frac{k_{half}}{2}$$

$$\Rightarrow \boxed{k_{half} = 2k}$$

Solution to problem 2 :

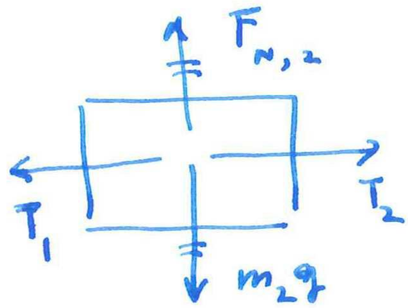
(a)

m_1 :



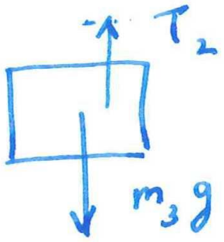
$(m_1 g = F_{N,1})$

m_2 :



$(m_2 g = F_{N,2})$

m_3 :



(b) method 1 :

$$m_1 a = T_1 \quad \ldots\ldots \ 1$$

$$m_2 a = T_2 - T_1 \quad \ldots\ldots \ 2$$

$$m_3 a = m_3 g - T_2 \quad \ldots\ldots \ 3$$

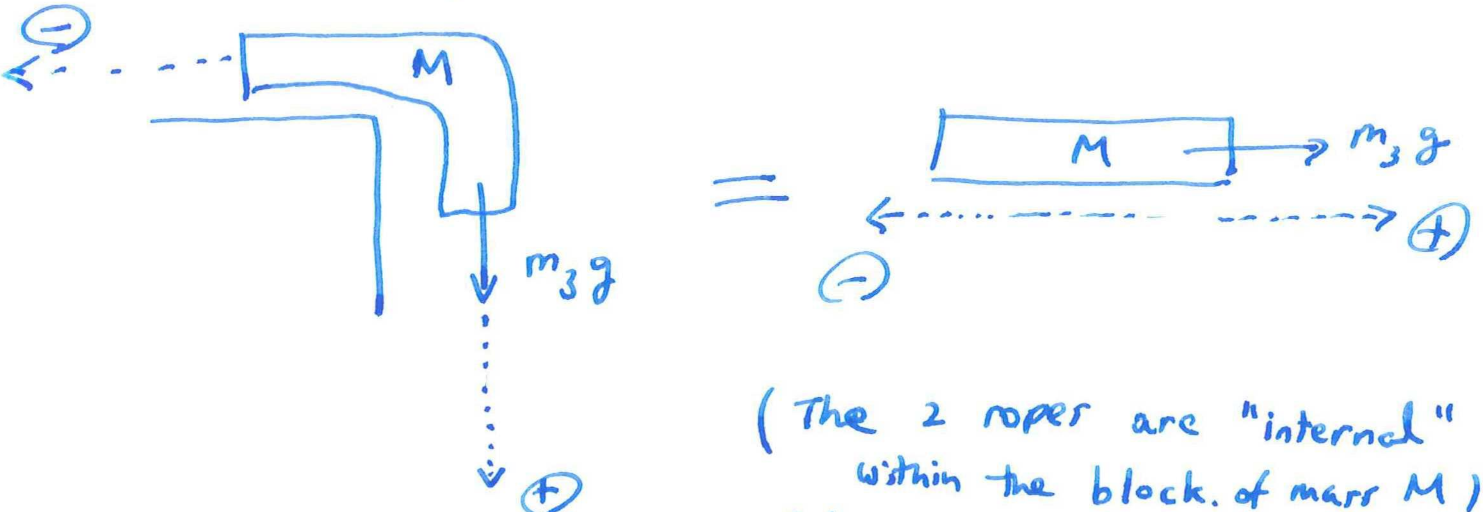
All three masses have the same acceleration a.

Add 1 + 2 + 3 :

$$a \cdot (m_1 + m_2 + m_3) = m_3 g$$

$$\Rightarrow \boxed{a = \frac{m_3 g}{m_1 + m_2 + m_3}}$$

method 2 : All 3 blocks move together as a single object of mass $m_1+m_2+m_3 \doteq M$



(The 2 ropes are "internal" within the block. of mass M)
Only the external forces matter.

so: $m_3 g = M a \Rightarrow \boxed{a = \dfrac{m_3 g}{m_1+m_2+m_3}}$

(c) $\boxed{T_1 = m_1 a = \dfrac{m_1 \, m_3 g}{m_1+m_2+m_3}}$

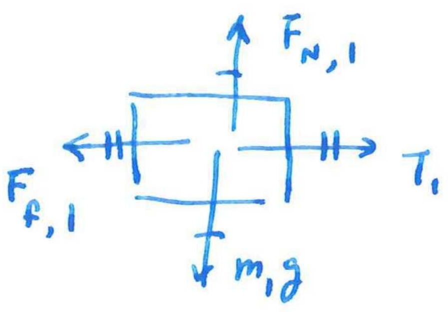
$T_2 = m_2 a + T_1$

$= (m_1+m_2)\, a$

$= \boxed{\dfrac{m_1+m_2}{m_1+m_2+m_3}\,(m_3 g)}$

(d) If everything is at rest, all forces must balance on each block :

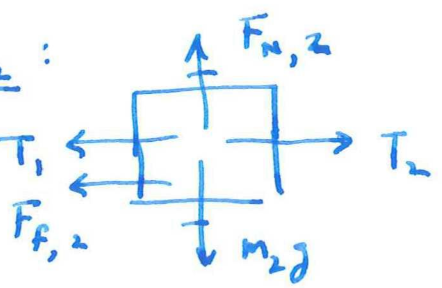
m_1 :



$T_1 = F_{f,1}$

$= \mu_s m_1 g.$

m_2 :



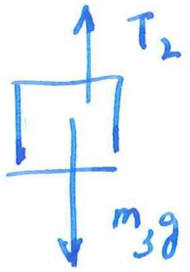
$T_2 = T_1 + F_{f,2}$

$= \mu_s m_1 g + \mu_s m_2 g$

$= \mu_s (m_1+m_2)\, g.$

Need

m_3 :



$$T_2 = m_3 g$$

$$\Rightarrow \mu_s (m_1 + m_2) \cancel{g} = m_3 \cancel{g}$$

$$\Rightarrow \boxed{\mu_s = \frac{m_3}{m_1 + m_2}}$$

At this value of μ_s, nothing moves.

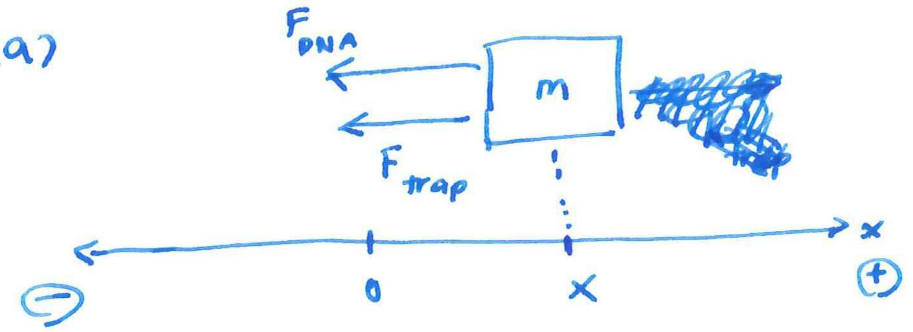
If μ_s is larger than this, then nothing moves.

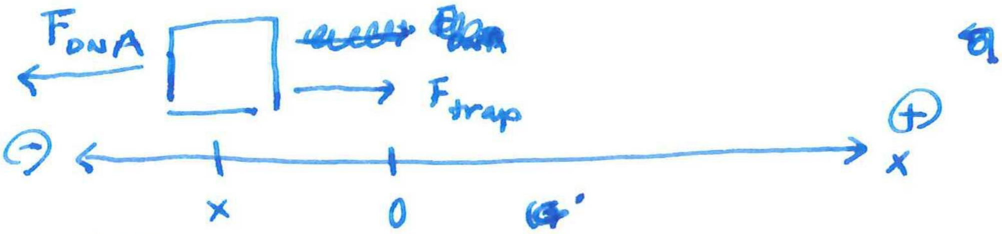
If μ_s is smaller than this, then blocks will move.

[so: $\boxed{\mu_s < \frac{m_3}{m_1 + m_2}}$ $\Rightarrow$ blocks will move.]

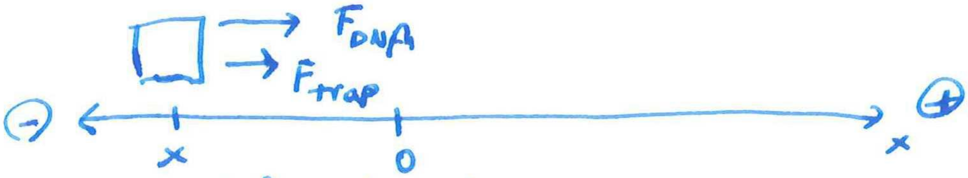
Solution to problem 3 :

(a)





(If $|x| < L$)



(If $|x| > L$)

Newton's 2nd law $\Rightarrow$ $$ma = F_{trap} + F_{DNA}$$

Sign convention:

$\ominus \longleftrightarrow \oplus$

$$F_{trap} = -k_{trap} x$$

$$F_{DNA} = -k_{DNA}(L+x)$$

$\}$ ← these match the direction of arrows in the pictures on previous page.

So: $$\boxed{ma = -k_{trap} x - k_{DNA}(L+x)}$$

(b) : Solution given after Quiz 2.

(c) : Solution given after Quiz 2.

(d) : From (b) : (see answer given on the question sheet if you didn't solve this problem yet.)

$E_{total} = PE + KE$ $\quad$ PE = potential energy in 2 springs

KE = kinetic energy

$$= \boxed{\frac{mv^2}{2} + \frac{L^2}{2}\left(k_{DNA}(1-\kappa)^2 + k_{trap}\kappa^2\right)}$$

(e) : When the block stops : KE = 0, but E_{total} same as before and is equal to entire potential energy.

$$E_{total} = \frac{k_{trap}}{2} x^2 + \frac{k_{DNA}}{2}(L+x)^2$$

$$\Rightarrow \quad 2E_{total} = (k_{trap} + k_{DNA})x^2 + 2k_{DNA}Lx + k_{DNA}L^2$$

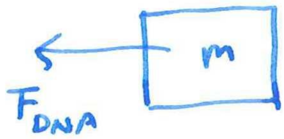
solve the quadratic equation to find X.

(you'll get 2 values for x.)

One for $X > 0$. ← where the block stops on right side

One for $x < 0$ ← where the block stops on left side.

(f) Right after the spring is cut:



$$F_{DNA} = -k_{DNA}(L + X_{eq})$$

x_{eq} = equilibrium position.

$$= -k_{DNA} L (1 - k)$$

$$k \equiv \frac{k_{DNA}}{k_{trap} + k_{DNA}}$$

↑ original spring constant of trap ~~before~~ before that spring was cut.

so $ma = F_{DNA}$

$$\Rightarrow \boxed{a = \frac{-k_{DNA} L (1-k)}{m}}$$

↳ acceleration (to the left due to the ⊖ sign.)

(g) $E_{total} = PE$

$$= \frac{1}{2} k_{DNA} (L + X_{eq})^2$$

$$= \boxed{\frac{1}{2} k_{DNA} L^2 (1 - k)^2}$$

(h) Before cutting; $E'_{tot} = \frac{k_{DNA} L^2}{2}(1-k)^2 + \frac{k_{trap} k^2 L^2}{2}$

so: $E'_{tot} - \underbrace{E_{tot}}_{\text{after cut}} = \frac{k_{trap} k^2 L^2}{2}$, ← "missing" energy

No, total energy in universe still the same.
The "lost" energy is due to the fact that ~~spring~~ the "trap spring" is no longer part of the system.
Our system now = DNA + bead.

But if we include the cut spring and the heat associated with the spring being cut, then we'll find that the total energy of universe remains conserved.

Solution to problem 4:

(a) $\vec{F} = -\frac{GMm}{r^2}\hat{r}$

$F = |\vec{F}| = \frac{GMm}{r^2}\underbrace{|\hat{r}|}_{=1 \text{ (unit vector)}} \Longrightarrow F = \frac{GMm}{r^2}$

$= \frac{GMm}{(R+h)^2} = \frac{GMm}{R^2\left(1+\frac{h}{R}\right)^2}$

(b) $f(u) \equiv \frac{1}{1+u}$

Then: $f(u) = f(0) + uf'(0) + \frac{u^2}{2!}f''(0) + \frac{u^3}{3!}f^{(3)}(0) + \ldots$

$= 1 + u\left(-\frac{1}{(1+u)^2}\right)\Big|_{u=0} + \frac{u^2}{2}\left(\frac{+2}{(1+u)^3}\right)\Big|_{u=0}$

$+ \frac{u^3}{6}\left(\frac{-6}{(1+u)^4}\right)\Big|_{u=0} + \ldots$

$= 1 - u + u^2 - u^3 + \ldots$

(c) $\left|\frac{\frac{u^2}{2}f''(0)}{uf'(0)}\right| = \left|\frac{u^2}{-u}\right| = |-u| = |u| = |h/R|$

To make $|u| \ll 1$, need to pick h so that $h \ll R$.

This is possible. (make h close to zero.)

(d) $\left|\frac{-u^3}{u^2}\right| = |u| \Rightarrow$ same reasoning as above.

(e) The ratio of 3^{rd} order to 1^{st} order is:

$$\left|\frac{u^3}{u}\right| = |u^2| \quad ; \text{ so for } |u| \ll 1 \text{ (i.e. } h \text{ close to zero.)}$$

we ~~have~~ have $|u^2| \ll |u|$.

e.g. pick $u = 0.01$

Then $u^2 = 0.0001$

so the 3^{rd} order term $(-u^3)$ is 100 times smaller in magnitude compared to 1^{st} order term u.

(f)
$$F = \frac{GMm}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$= \frac{GMm}{R^2} \cdot \frac{1}{(1+u)^2} \qquad u = h/R$$

$$\approx \frac{GMm}{R^2} \cdot (1-2u)$$

$$= \boxed{\frac{GMm}{R^2}\left(1-2h/R\right)}$$

(g) $\underline{h=0} \Rightarrow F = \frac{GMm}{R^2} \Rightarrow \boxed{\frac{F}{m} = \frac{GM}{R^2} = g = 9.8\ m/s^2}$