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Physics 1A

2017/1/24

Chapter 18: Heat, Work, and the 1st law of thermodynamics

- Conservation of energy:

  is a quantitative statement for mechanical energy in the presence of conservative forces.

- Non-conservative forces convert mech. energy into <u>random</u> molecular energy (called internal energy)

- In c. 16 and 17 we learned that thermal processes involve energy.

* The extention of conservation of energy in thermodynamic systems is the 1st law of thermodynamics.

Why is heat a form of energy?

Want to raise T of water in a container

Heat it ↙

Stir it vigorously ↘

- spoon does mech. work on water
- Doing work increases mech. energy of a macroscopic object

But here:

- Stirring changes the int. energy, E_{int}, associated w/ molecules

↘ ↙

both ways lead to the same final state of water

⇓

Higher temp. ⇒ Higher int. energy

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The changes in the internal energy of water depends only on the net energy transfered

↓

This is a statement of cons. of energy (note that "heat" is now included in this statement)

↓

1st law of thermodynamics

The change in the internal energy of a system depends only on the net heat transferred to the system and the net work done on the system independent of the particular processes involved

↓ Mathematically

$$\boxed{\Delta E_{int} = Q + W}$$

ΔE_{int} ↑ net change in a system's int. energy

Q ↑ heat transferred to the system

W ↑ work done ON the system

Side note

E_{int} is a thermodynamic state variable

↑ a quantity whose value does not depend on how a system got into its particular state (e.g. temp. and pressure)

(heat and work are not therm. state variables)

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Often we care about rates and not the actual value of E_{int}, Q, and W :

$$\Delta E_{int} = W + Q \implies \frac{dE_{int}}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}$$

Note

1^{st} law of ther. applies to any system (it's universal) but it's easier if use it for an ideal gas

$$\underline{\text{ideal gas}} \downarrow \quad pV = nRT$$

Recall

$$pV = NKT \underset{\underset{N=nN_A}{\uparrow}}{=} nN_AKT \underset{\underset{N_AK=R}{\uparrow}}{=} nRT$$

$N \equiv$ # of molecules in the gas

$n \equiv$ # of moles

$N_A \equiv$ Avogadro's number (atoms or molecules)

$R \equiv$ Universal gas constant

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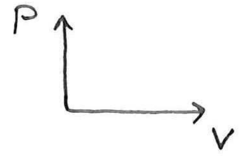
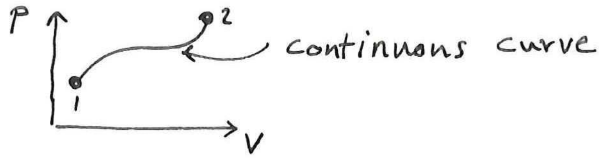
PV diagram

- The thermodynamic state of a system is completely determined by 2 of $p, V,$ and T.
- Can represent states as points on a PV diagram



A quasi-static process

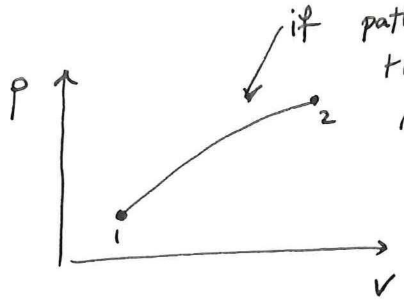
- when the system (gas) remains in equilibrium w/ the reservoir it's in (both have the same temp.) while the temp. changes slowly



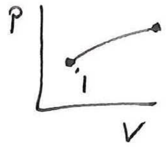
- we could always reverse the path $1 \rightarrow 2$ to get $2 \rightarrow 1$ for a QSP. QSP is a reversible process

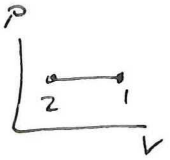
- For an irreversible process, sys. is not in equil. and variables like T and p don't have a well defined relationship.

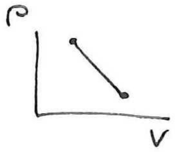
The path matters not the end points.

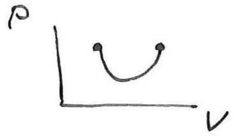


if path is continuous the the process is reversible we can think of all sorts of processes in a pV diagram.





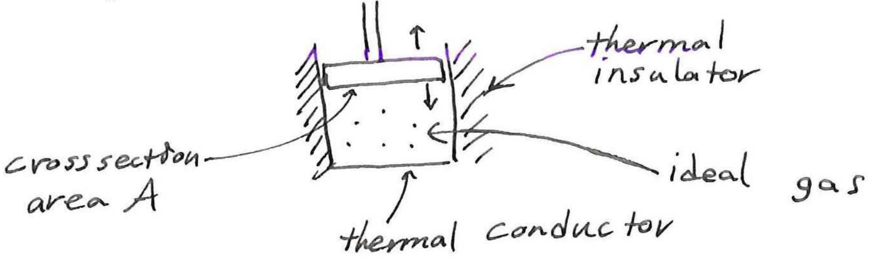




but we're going to focus on a few special cases (all involving ideal gas)

before getting there :

- First: we need a system that allows us to vary P, V, T of an ideal gas



we can change the thermodynamic state of ideal gas inside by :

a. mechanically: moving the piston up or down

b. thermally: heating the gas from the bottom

- Second: need to calculate the work done <u>on</u> the gas which holds for all processes

the force that the gas inside exerts on the piston is

$$F_{\text{gas}} = PA$$

Recall that $W = \vec{F}\cdot\vec{d}$

thus if the piston moves a small distance ΔX, then

$$\Delta W_{\text{gas on piston}} = F_{\text{gas}}\,\Delta X = \underbrace{PA}\,\underbrace{\Delta X} = P\underset{\substack{\uparrow\\ \text{change in gas}\\ \text{volume}}}{\Delta V}$$

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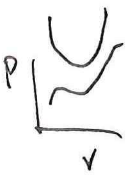
Recall that 1st law was

$$\Delta E_{int} = Q + \underset{\substack{\uparrow \\ \text{work done on gas}}}{W}$$

by Newton's 3rd law $W_{\substack{\text{done}\\\text{on}\\\text{gas}}} = - W_{\substack{\text{done}\\\text{by}\\\text{gas}}}$

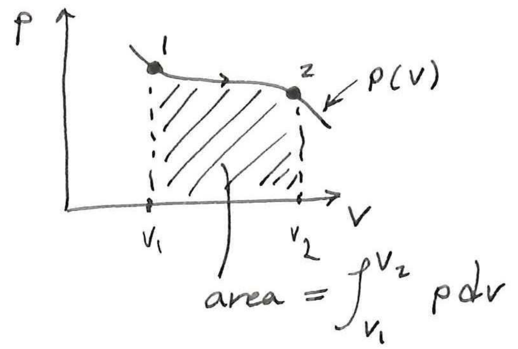
$$\Delta W_{\substack{\text{done}\\\text{on}\\\text{gas}}} = - F_{gas}\,\Delta x = - P\,\Delta V \quad \leftarrow \quad \Delta W_{\substack{\text{done}\\\underline{\underline{\text{on}}}\\\text{gas}}} = - P\,\Delta V$$

but since P <u>can</u> change as V changes,



$$\underset{\substack{\uparrow \\ \text{work done on gas}}}{W} = \int dw = - \int_{V_1}^{V_2} \underset{\substack{\nwarrow \\ p(v)}}{p}\, dV$$

as vol. changes from V_1 to V_2



$$\text{area} = \int_{V_1}^{V_2} p\,dv$$

$$\left\{ \begin{array}{ll} W > 0 & \text{if } V_2 < V_1 \text{ (gas is compressed)} \\ W < 0 & \text{if } V_2 > V_1 \text{ (gas expands)} \end{array} \right.$$

↑ done on gas

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// special processes in PV diagram

↑ continuous ~~processes~~ transitions between thermodynamic states

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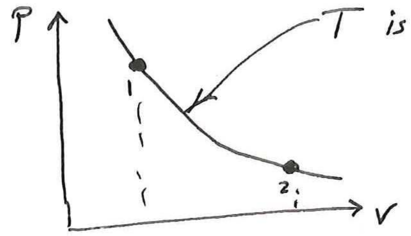
1. Isothermal processes ⟺ constant temp.



T is the same everywhere on this line (isotherm)

$$W_{\substack{\text{on}\\\text{gas}}} = -\int_{V_1}^{V_2} P(V)\,dV = -\int_{V_1}^{V_2} \frac{nRT}{V}\,dV = -nRT\int_{V_1}^{V_2} \frac{1}{V}\,dV =$$

$PV = nRT$

$$= -nRT \ln V \Big|_{V_1}^{V_2} = -nRT\left(\ln V_2 - \ln V_1\right) = -nRT \ln \frac{V_2}{V_1}$$

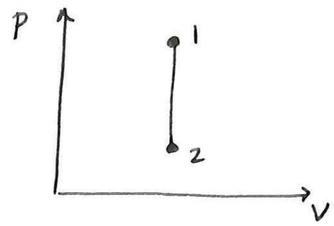
Important

int. energy of a gas depends only on → kinetic energy of its molecules which depends only on → temperature

$$\Rightarrow \Delta E_{int} = 0 \Rightarrow 1^{st} \text{ law: } \boxed{0 = Q + W}$$

$$\boxed{Q = -W = nRT \ln\left(\frac{V_2}{V_1}\right)}$$

2. Constant Volume process + specific heat C_V



$V_1 = V_2$

no piston movement ⟶ no work done on or by gas

$$\Rightarrow 1^{st}\text{ law}: \quad \Delta E_{int} = Q + \underset{0}{\cancel{W}}$$

Can represent Q as a temperature change ΔT :

$$Q = n\,C_V\,\Delta T$$

n: # of moles

C_V: molar specific heat at cons. vol. (similar to specific heat in ch.16 but is per mole instead of per unit mass)

Recall ch.16.2

$$Q = C\,\Delta T$$

C: heat capacity (J/K)

$C = cm$

$$\begin{cases} \Delta E_{int} = Q \\ Q = nC_V\Delta T \end{cases} \Longrightarrow \boxed{\Delta E_{int} = n C_V \Delta T}$$

<u>Important</u>

for ideal gas, int. energy is <u>only</u> a function of temp. and hence $\frac{\Delta E_{int}}{\Delta T} = nC_V$ has the same value no matter what process

* specific heat measures the heat needed to cause a given temp. change

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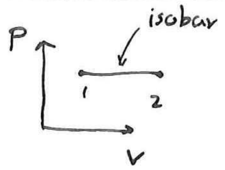
3. Isobaric Processes + Specific Heat C_p



e.g. boiling water for cooking pasta

since p is constant W is easy to find

$$W = -\int_{V_1}^{V_2} p\,dV = -p(V_2 - V_1) = -p\,\Delta V$$

$$Q = \Delta E_{int} - W = \Delta E_{int} + p\,\Delta V$$

$$\Delta E_{int} = n C_v \Delta T$$

$$Q = n C_v \Delta T + p \Delta V$$

Define

$$Q = n C_p \Delta T \quad \text{(for isobaric processes)}$$

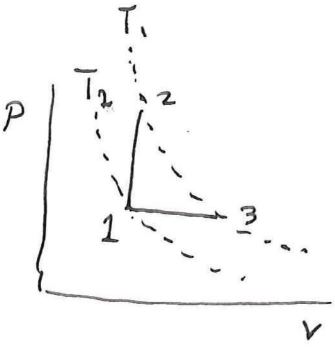
$\uparrow$ molar specific heat at constant pressure

$$\rightarrow \quad n C_p \Delta T = n C_v \Delta T + p \Delta V \quad \longleftarrow \text{only for isobaric processes}$$

this is a useful equation for calculating ΔT if we know C_p and C_v. However, we only need one of them.

for ideal gas $pV = nRT$ can write $p\Delta V = nR\Delta T$

$$n C_p \Delta T = n C_v \Delta T + n R \Delta T \implies \boxed{C_p = C_v + R}$$



$\longleftarrow$ This tells us that a constant P process requires more heat for a given temp change than a constant volume process.

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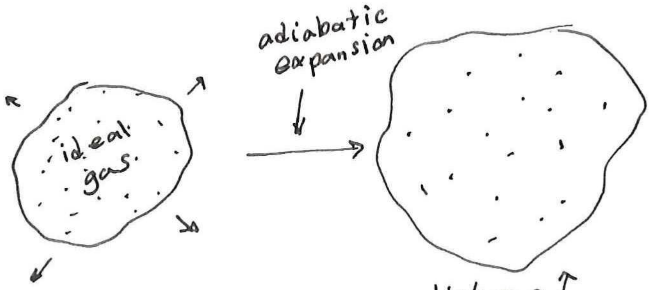
4. Adiabatic processes

For this process, no heat flows between a system and the environment it's in. ⇒ $Q = 0$

Two ways:
1. complete insulation of the system
2. for the process to be so quick that it's over before heat has had time to flow (e.g. in gasoline engines: both the compression of gas/air mix and the expansion of the combustion product are almost adiabatic)

$\boxed{\Delta E_{int} = Q + W}$

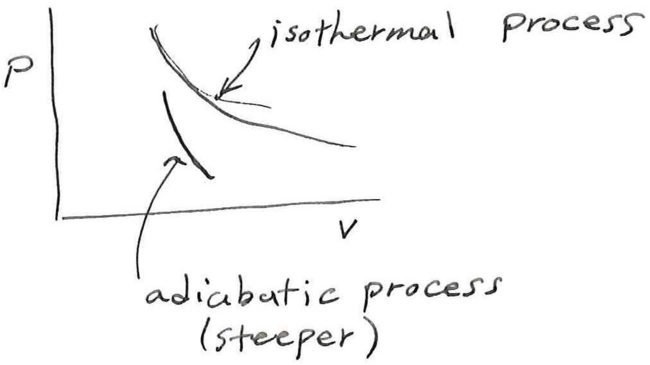
1st law → $\Delta E_{int} = W$



Volume ↑
int energy ↓
temp ↓
P ↓

$pV = nRT$

* therefore pressure drop is greater in adiabatic processes than in isothermal ones.



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$$\Delta E_{int} = nC_v\Delta T \longrightarrow dE_{int} = nC_v\,dT$$

$$\Delta W = p\Delta V \longrightarrow dW = p\,dv$$

for adiabatic processes where $Q=0$, 1st law

$$dE_{int} = dW \longrightarrow nC_v\,dT = -p\,dv$$

$$\boxed{\begin{aligned} pV = nRT \rightarrow\ & nR\,dT = d(pV) = dp\cdot v + p\,dv \\ & \Rightarrow dT = \frac{p\,dv + v\,dp}{nR} \end{aligned}}$$

$$nC_v\,\frac{p\,dv + v\,dp}{nR} = -p\,dv \longrightarrow C_v(p\,dv + v\,dp) = -pR\,dv$$

$$\rightarrow C_v p\,dV + C_v V\,dp + pR\,dV = 0$$

$$C_v V\,dp + (C_v P + pR)\,dV = 0$$

$$C_v V\,dp + (C_v + R)\,p\,dV = 0$$

$$C_p = C_v + R$$

$$C_v V\,dp + (\cancel{C_v} + C_p - \cancel{C_v})\,p\,dv = 0$$

$$C_v V\,dp + C_p\,p\,dV = 0$$

$$\times\frac{1}{C_v PV}$$

$$\frac{dp}{p} + \frac{C_p}{C_v}\,\frac{dv}{V} = 0$$

$$\gamma \equiv \frac{C_p}{C_v}$$

$$\int\frac{dp}{p} + \int\gamma\,\frac{dv}{V} = \text{Const.} \Longrightarrow \ln(p) + \gamma\ln(V) = \text{constant}$$

$$\Longrightarrow \cancel{pV^\gamma} \Longrightarrow \boxed{pV^\gamma = \text{constant}}$$

for adiabatic processes

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$$PV^{\gamma} = c$$

$$PV = nRT \longrightarrow P = \frac{nRT}{V}$$

$$\frac{nRT}{V} V^{\gamma} = c \longrightarrow TV^{\gamma-1} = \frac{c}{nR} = c' \Longrightarrow TV^{\gamma-1} = \text{constant}$$

$$W = -\int_{V_1}^{V_2} p\,dV = -\int_{V_1}^{V_2} c\,\frac{dV}{V^{\gamma}} = -c\,\frac{\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)}{1-\gamma}$$

$$\boxed{\int \frac{dx}{x^{a}} = \frac{x^{1-a}}{1-a} + \text{constant}}$$

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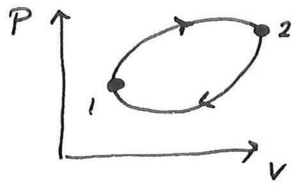
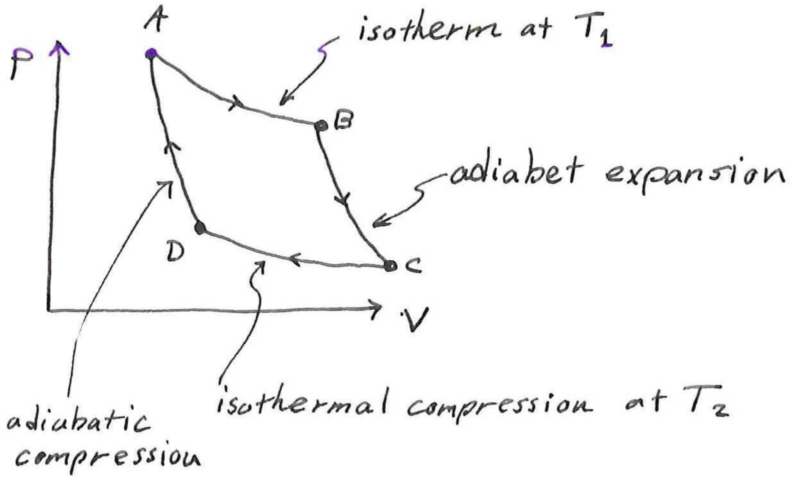
Cyclic Processes



- Requires both expansion and compression
- Internal energy of a system depends <u>only</u> on the thermodynamic state of that system, therefore, for a cyclic process the net change of internal energy is zero:

$$\Delta E_{int} = 0 \quad \Longrightarrow \quad Q_{net} = -W_{net}$$

Carnot Cycle



⇐ this is the PV-diagram of an ideal gas going through what's called a "Carnot Cycle"

Process A → B

- isothermal expansion at constant temperature T_1
- During any isothermal process $\Delta E_{int} = 0$

Therefore, 1st law for process A → B is:

$$\underset{0}{\cancel{\Delta E}}{}_{\substack{int\\ A\to B}} = Q_{AB} + W_{AB} \quad \Longrightarrow \quad Q_{AB} = -W_{AB}$$

Heat transferred to the gas during process A→B

Work done <u>on</u> the gas during process A→B

$$W_{AB} = -\int_{V_A}^{V_B} p\,dv = -nRT_1 \ln\left(\frac{V_B}{V_A}\right)$$

← this tells us that the work done on the gas is negative

But since $Q_{AB} = -W_{AB}$ then the heat transferred to the gas is positive and it's equal to $nRT_1 \ln\left(\frac{V_B}{V_A}\right)$

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Process B → C

- Adiabatic expansion
- During an adiabatic process $Q = 0$, therefore, for here:

$$Q_{BC} = 0$$

which means no heat is exchanged between the gas and it's surrounding.

1st law:

$$\Delta E_{\substack{int \\ B\to C}} = \cancelto{0}{Q_{BC}} + W_{BC}$$

Also we know that for <u>any</u> process $\Delta E_{int} = nC_V \, \Delta T$

and therefore, $\Delta E_{\substack{int \\ B\to C}} = nC_V \, \Delta T_{BC} = nC_V (T_2 - T_1)$

$$\Rightarrow \left.\begin{aligned} \Delta E_{\substack{int \\ B\to C}} &= W_{BC} \\ \Delta E_{\substack{int \\ B\to C}} &= nC_V (T_2 - T_1) \end{aligned}\right\} \Rightarrow W_{BC} = nC_V (T_2 - T_1)$$

In class, I mistakenly put a negative sign here. One of your classmates pointed out my mistake.

Process C→D.

- Isothermal compression $\Delta E_{\underset{C\to D}{int}} = 0$
- Positive work is done on the gas

$-\ln\left(\frac{a}{b}\right) = \ln\left(\frac{b}{a}\right)$

1st law: $Q_{CD} = -W_{CD}$

$$W_{CD} = -\int_{V_C}^{V_D} p\,dV = -nRT_2 \ln\left(\frac{V_D}{V_C}\right) = nRT_2 \ln\left(\frac{V_C}{V_D}\right)$$

$V_C > V_D$

Work done ~~on~~ on gas is positive

$$Q_{CD} = -W_{CD} = -nRT_2 \ln\left(\frac{V_C}{V_D}\right)$$

this negative sign tells us that $nRT_2 \ln\left(\frac{V_C}{V_D}\right)$ is the amount of heat that is leaving the gas

Process D→A

- Adiabatic compression → $Q_{DA} = 0$

$$\Delta E_{\underset{D\to A}{int}} = \cancelto{0}{Q_{DA}} + W_{DA}$$

$$\Delta E_{\underset{D\to A}{int}} = nC_V \Delta T_{DA} = nC_V(T_1 - T_2)$$

$$\Bigg\} \; W_{DA} = nC_V(T_1 - T_2)$$

Net heat absorbed by the gas:

$$Q_{\underset{absorbed}{net}} = Q_{AB} + Q_{CD} = nRT_1 \ln\left(\frac{V_B}{V_A}\right) + \left(-nRT_2 \ln\left(\frac{V_C}{V_D}\right)\right) =$$

$$= nR\left[T_1 \ln\left(\frac{V_B}{V_A}\right) + T_2 \ln\left(\frac{V_D}{V_C}\right)\right]$$