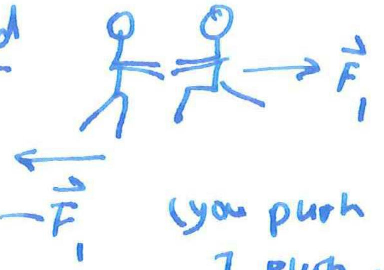
yesterday: Describing $\vec{r}(t)$, $\vec{v}(t)$, $\vec{a}(t)$.

Today: what makes things move = force.

- Newton's law stated simply: $\vec{F} = m\vec{a}$.

and

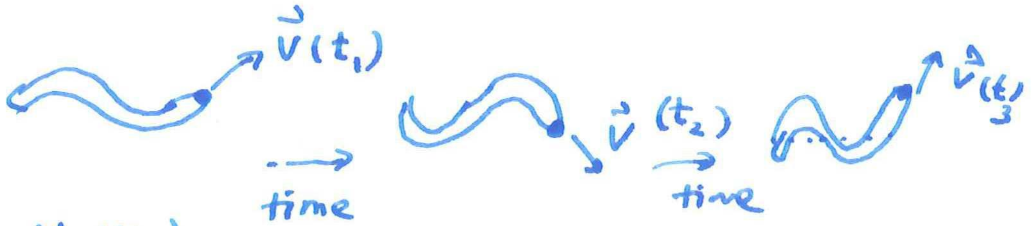


(you push me, I push you back.)

1) c. elegan's avg. velocity,

- from yesterday:

c. elegan's head



$$\vec{v}(t) = (v_x(t),\ v_y(t))$$

$$= (v_0,\ v_0\cos(\omega t))$$

$$\vec{r}(t) = \left(v_0 t,\ \frac{v_0}{\omega}\sin(\omega t)\right)$$

Qu: Watch c. elegan move for a long time.

~~what is would you~~ What is the average velocity of the worm's head?

Sol'n: Say we observe worm from $t=0$ to $t=T$

~~tht~~ Want $\vec{v}_{avg} = (v_{x,avg},\ v_{y,avg})$

constant vector.

- Avg. vel. means, I don't want to think about a velocity that's changing over time. Let's simplify by using a constant vector.

$$\underbrace{\vec{r}(T) - \vec{r}(0)}_{\text{Displacement vector.}} = \vec{V}_{avg.}\, T.$$

$\|$ y

$$\left(V_0 T, \frac{V_0}{\omega}\sin(\omega T) \right) = \left(V_{x,avg} T, \; V_{y,avg} T \right)$$

x-comp. : $V_{x,avg} T = V_0 T \Rightarrow \boxed{V_{x,avg} = V_0}$

y-comp : $\frac{V_0}{\omega}\sin(\omega T) = V_{y,avg} T \Rightarrow \boxed{V_{y,avg} = 0}$

Pick T so that $\omega T = 2\pi, 4\pi, \ldots\ldots 2n\pi$

$(n = 1, 2, 3, \ldots\ldots)$

(long time means that we pick T large and a "nice value")

· Lecture note does this differently. Look at it.

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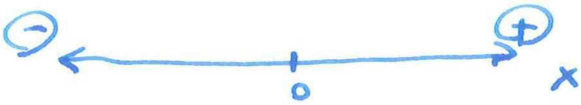
2) E. coli Chemotaxis :



Case 1 : No food

$\bullet\!\longrightarrow$ v for time ΔT.

$\longleftarrow\!\bullet$ v for time ΔT

$\bullet\!\longrightarrow$ v for time ΔT

⋮ and so on.

Watch for a long time. What's the average velocity?

sol'n: Let's pick $t_{total} = NT$ (or $t_{tot} = nT + nT$) | L2-3

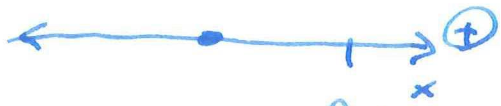
$V_{avg}(NT) =$ total displacement

$= V\underbrace{(N/2)}_{n}T + (-V)\underbrace{(N/2)}_{n}T$

or:

$V_{avg} = 0.$

Case 2: Say



find $\langle x \rangle$

$\xrightarrow{V}$ for time $t = kT$ $(k>1)$

$\xleftarrow[V]{}$ T

$\xrightarrow[V]{}$ for time kT

⋮

and so on.

pick. ~~time~~ $\in NT$

~~N = large even #.~~

$t_{total} = nkT + nT.$

~~$V_{avg}(NT) = V(kT)(N/2) + (-V)T$~~

$V_{avg}\, t_{total} = VkTn + (-V)nT$

$\Rightarrow V_{avg} = \dfrac{\cancel{n}\cancel{T}V[k-1]}{\cancel{n}k\cancel{T} + \cancel{n}\cancel{T}} = \dfrac{V\cdot(k-1)}{k+1}$

So if $k=2$: $V_{avg} = \dfrac{V}{3}.$

Also: if $k \gg 1$: $V_{avg} \approx V.$

if $0<k\ll 1$:

(extreme case: $k=0$: $V_{avg} = -V.$

mistake in my notes!

$$f(k) = \frac{1}{1+k}$$

by Taylor approx:

$$f(k) \simeq f(0) + k f'(0) + \frac{k^2}{2} f''(0) + \ldots\ldots$$

$$= 1 + k\left(-\frac{1}{(1+k)^{2}}\Big|_0\right) + \frac{k^2}{2}\underbrace{\#}_{a_2} + \frac{k^3}{3!}\underbrace{\#}_{a_3} + \ldots$$

$$= 1 - k + \frac{k^2}{2}a_2 + \frac{a_3}{3!}k^3 + \ldots$$

Say $k = 0.01$:

$k^2 = 0.0001$ ← 100 times smaller than k

so $k + k^2 = 0.0101 \sim k$.

so: $\cancel{f(k) \simeq}$ $\frac{1}{1+k} \simeq 1-k$.

so:

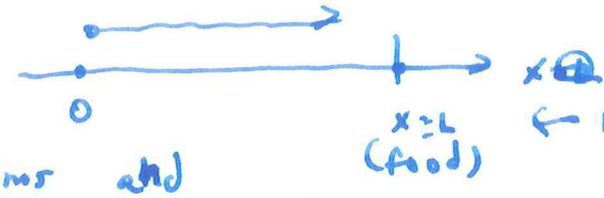
$$V_{avg} \simeq \frac{V(k-1)}{k+1} \simeq V(k-1)(1-k)$$

$$= -V(1-k)^2$$

$$\simeq -V(1-2k)$$

$$V_{avg} = \boxed{-V + 2kV}$$

(c) Now:



← reaches here at $t = T$.

swims and reaches the food: $V(t) = V_0 e^{t/T}$

$(0, t_1)$, (t_1, t_2), ..., (t_{N-1}, t_N) ← N segments of time.

$V(0)$ $V(t_1)$ ..... $V(t_{N-1})$

Δt Δt ..... Δt

$N\Delta t = T$

$$V_{avg.} T = V(0)\Delta t + V(t_1)\Delta t + \dots V(t_{N-1})\Delta t$$

$$V_{avg} = \frac{1}{T}\sum_{i=1}^{N} V(t_{i-1})\Delta t \leftarrow dt$$

$$= \frac{1}{T}\int_0^T V\, dt$$

$$= \frac{1}{T}\int_0^T V_0 e^{t/T}\, dt$$

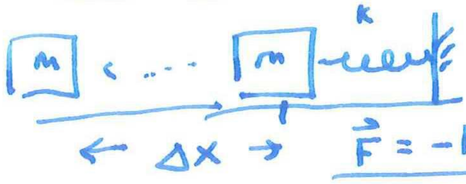
$$= \boxed{V_0(e-1)}$$

· forces: In the book

① · Draw force diagrams. ← break forces into parts.

② ~~[illegible]~~ [illegible]

spring force:



$\vec{F} = -k\Delta x$.

③ friction

④ gravity:

mg.

① Always draw a force diagram.

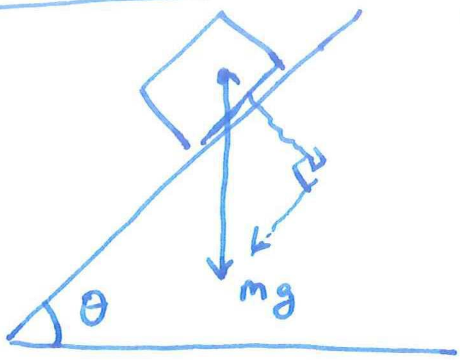
$$\vec{F} = m\vec{a} \Rightarrow (F_x, F_y) = (ma_x, ma_y)$$

vectors.

so can take care component by component.

(find which direction you're interested in).

· friction + components
gravity



↓ g : $g = 9.8\ m/s^2$

(⊕ #)

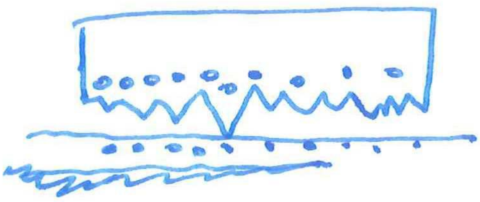
2-6

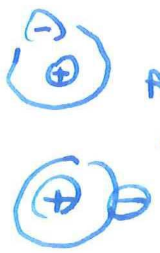
say ∃ kinetic friction : friction when trying / is moving

· what are all the forces?

friction occurs
∵ roughness of surface
& adhesion resists ~~from~~
the block's movement.

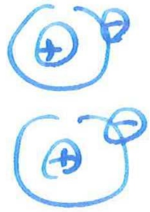
why friction :





far apart
(don't see each other)
They look neutral

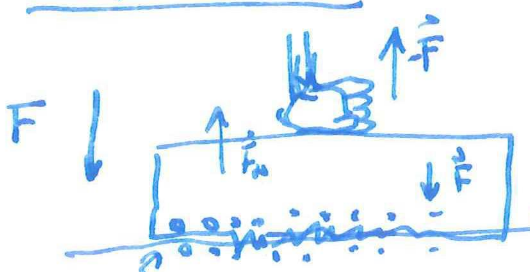
closer together :



Now they see each other.
∃ attraction.

(molecular bond).

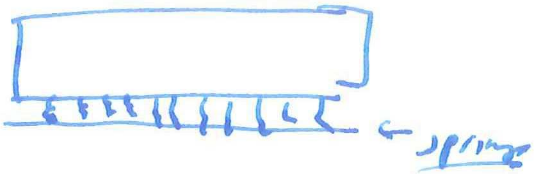
Now, you push down :

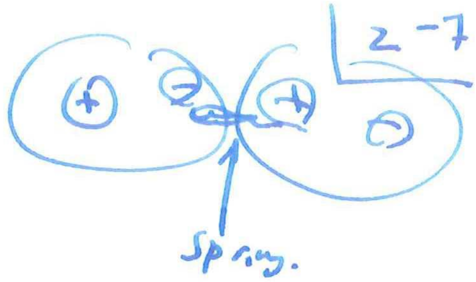


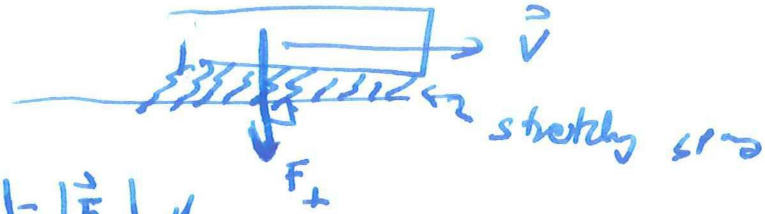
squash & compress

molecules see each other ⇒ more bond between floor and object

(More you press down, ∃ more bond)

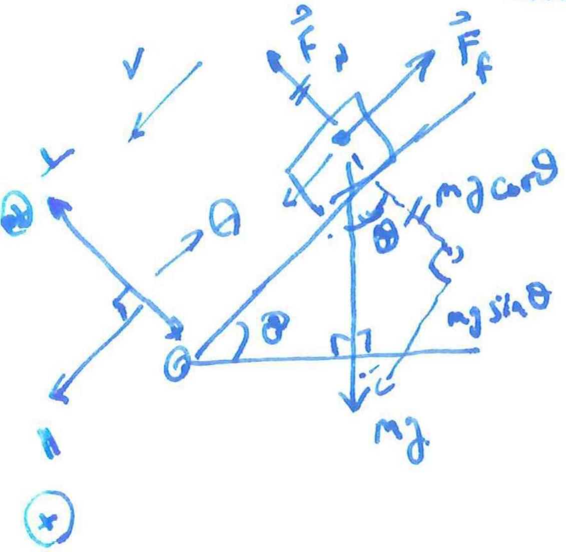






$$|\vec{F}_f| = |\vec{F}_\perp| \mu_k$$

coeff kinetic friction.



∥-direction:

$$mg\sin\theta - F_f = m a_\parallel$$

$$F_N = mg\cos\theta \quad : \quad F_N - mg\cos\theta = 0.$$

$$F_f = \mu_k F_N$$

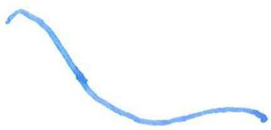
$$\Rightarrow F_f = \mu_k mg\cos\theta > 0.$$

$$\therefore \quad mg\sin\theta - \mu_k mg\cos\theta = m a_\parallel$$

$$\Rightarrow \boxed{a_\parallel = g\sin\theta - g\mu_k\cos\theta}$$

expect $a_\parallel > 0$.

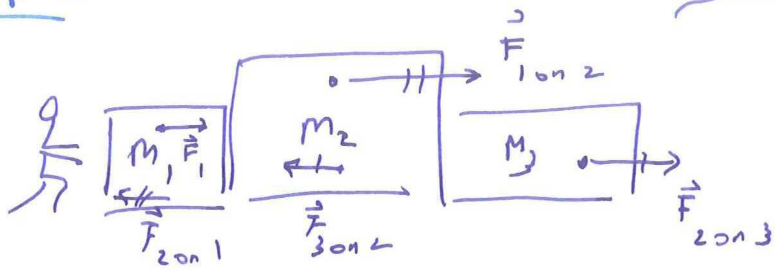
ex



~~Spring force~~

No friction

e.g



$\vec{F}_{1 on 2} = -\vec{F}_{2 on 1}$

$\vec{F}_{3 on 2} = -\vec{F}_{2 on 3}$

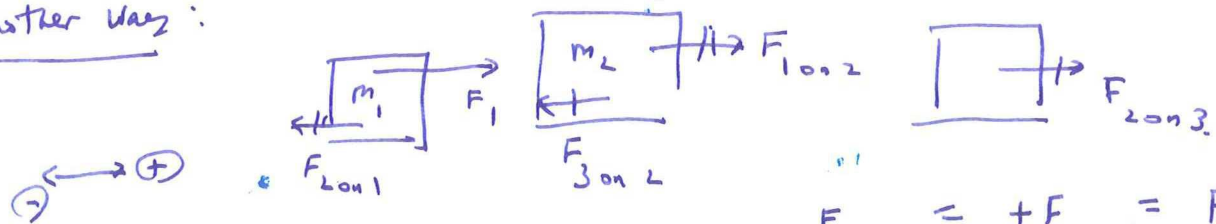
- you are constantly pushing at force $\vec{F}$.
- what's the acceleration of the 3 blocks?

- They are all moving together as one giant block

$$(m_1 + m_2 + m_3)\vec{a} = \vec{F}_{tot} = \vec{F}_1$$

$$\boxed{\vec{a} = \frac{\vec{F}_1}{m_1 + m_2 + m_3}}$$

Another way:



$F_{1 on 2} = +F_{2 on 1} = F_2$

~~[illegible]~~ + #'s

$F_{2 on 3} = F_{3 on 2} = F_3$

+ #

$m_1 a = F_1 - F_2$ ..... ①

$m_2 a = F_2 - F_3$ ..... ②

$m_3 a = F_3$ ..... ③

Add all 3 equations

$$m_1 a + m_2 a + m_3 a = F_1$$

$$\Rightarrow \boxed{a = \frac{F_1}{m_1 + m_2 + m_3}}$$

$$\Rightarrow \boxed{F_3 = \frac{m_3}{m_1 + m_2 + m_3} F_1}$$

$$F_2 = F_3 + m_2 a$$

$$= \frac{m_3}{m_1 + m_2 + m_3} F_1 + \frac{m_2 F_1}{m_1 + m_2 + m_3}$$

$$\Rightarrow \boxed{\frac{(m_2 + m_3)}{m_1 + m_2 + m_3} F_1}$$

← makes sense too!