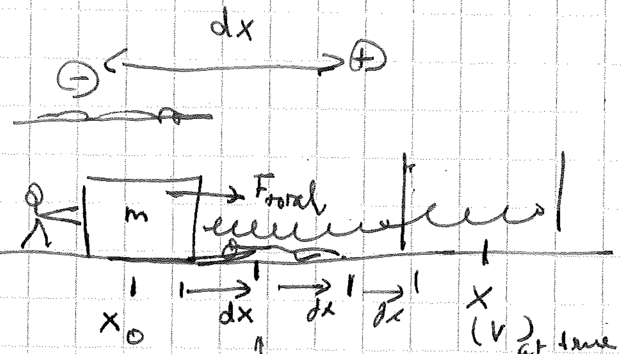
w

Work, kinetic energy, potential energy

In 1dimearm motion

Consider Newton's law (9n



(At t=0) some later time dt

(V_0)

$$F_{total} = ma$$
$$= m\frac{dV}{dt}$$

$$F_{total}\,dx = m\frac{dV}{dt}dx$$
$$= m\,dV\,\frac{dx}{dt}$$
$$= mV\,dV$$

$$\int_{x_0}^{x} F_{total}\,dx = \int_{V_0}^{V} mV\,dV$$

$$W = \frac{mV^2}{2} - \frac{mV_0^2}{2}$$

$$\Rightarrow \boxed{\frac{mV^2}{2} - W = \frac{mV_0^2}{2}}$$

↑ changes over time during motion. ↑ Constant.

Another way.

$$\int F_{tot}\,dx = \int mV\,dV$$
$$W = \frac{mV^2}{2} + \text{Const.}$$

$$\Rightarrow \boxed{-\text{Constant} = \frac{mV^2}{2} - W.}$$

We call $W_{x_0\to x} \equiv \int_{x_0}^{x} F_{total}\,dx$ ← work

$-W_{x_0\to x} \equiv$ potential energy $\equiv \underline{U}$

$KE \equiv \frac{mV^2}{2}$

↑ from this we get all possible relationships

(1) $W_{x_0\to x} = \frac{mV^2}{2} - \frac{mV_0^2}{2}$ >0 means you put in energy

total work done ↗ ↑ change in KE from t=0 to late time t

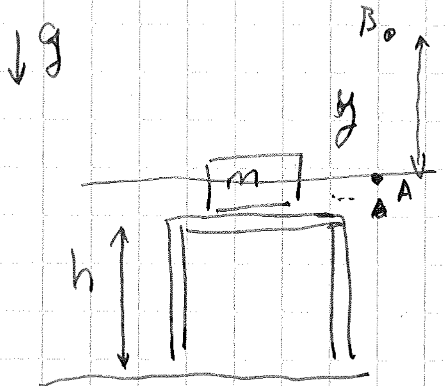
(2) $\frac{mV^2}{2} - W = \frac{mV^2}{2} + U. =$ Constant ← conservation of energy.

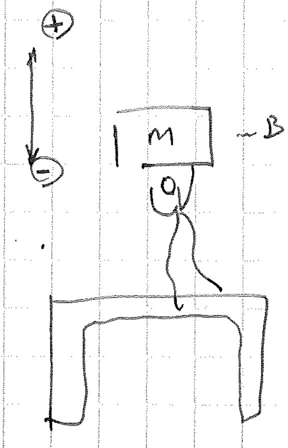
↑ doesn't change over time

(3) $-\frac{dU}{dx} = F$ ← only for special cases. (Assumes U can be written as funct of x)

e.g. Constant Gravity

Work that you do $\neq$ total work on the system





~~What's the~~

~~How much work did you do?~~

$W_{A\to B} =$

suppose you release the block: what is the speed afterwards? ~~in~~ when it reaches point A.?

$-$ y, mg, $+$

$$W_{B\to A} = \frac{mv_A^2}{2} - \frac{mv_B^2}{2}$$

$$= \int_{y_B}^{y_A} mg\,dy$$

$$mg(y_A - y_B) = \frac{mv_A^2}{2} \Rightarrow \boxed{mg\,\Delta y = \frac{mv_A^2}{2}}$$

so $y_A > y_B$

If sign reversed

$+$, $-$, mg

$$\int_{y_B}^{y_A} F\cdot dy \Longrightarrow \text{still get } \boxed{mg\,\Delta y = \frac{mv_A^2}{2}}$$

$$= \int_{y_B}^{y_A} (-mg)\,dy$$

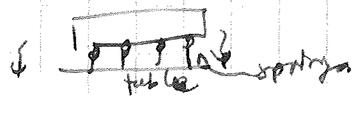
$$= -mg\,(y_A - y_B)$$

< 0

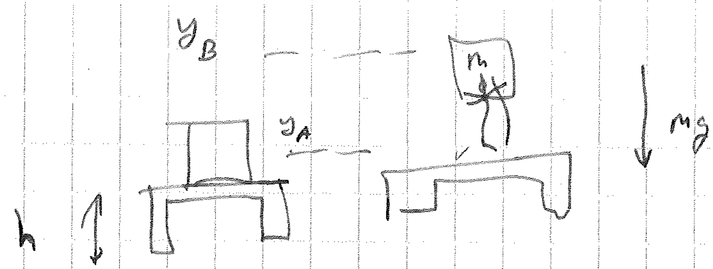
$\because y_A < y_B$.

· After it hits the table: ~~KE=0~~ $\frac{mv_A^2}{2} - W_{BA} = \frac{mv_B^2}{2} \Rightarrow$ so $W_{grav.} = 0$??

No, it's that the table did work



· In the gravity example:



$W = \int F_{tot}\, dx$

$= \int$

Qu: · How much work did you do in going from y_A to y_B?
· What's the total work done on the block?

sol'n

$$W_{you} = \int_{y_A}^{y_B} F\, dy = mg\,(y_B - y_A)$$

$$W_{grav} = \int_{y_A}^{y_B} (-mg)\, dy = -mg\,(y_B - y_A)$$

(+) (−) $\downarrow -mg$ $\uparrow mg$

so $\boxed{W_{you} + W_{grav} = 0}$

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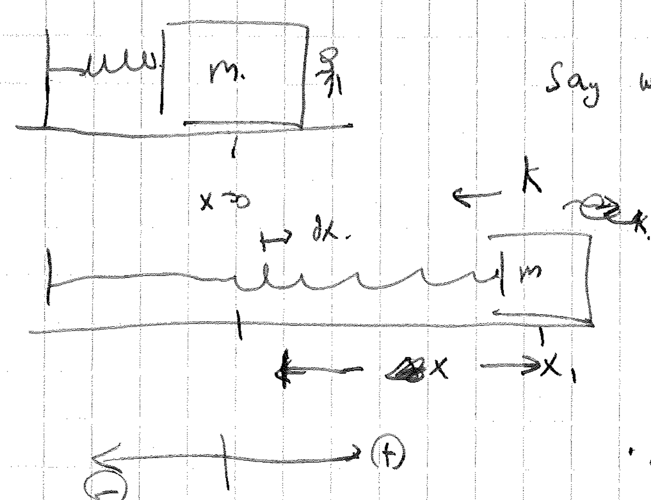
ex: Spring system (force not constant anymore).



Say we stretch

$F_{spring} = -kx$.

Qu: How much work did you do in stretching the spring?

· Again $W_{tot} = 0$ ∵ no KE at the end and no KE at beginning

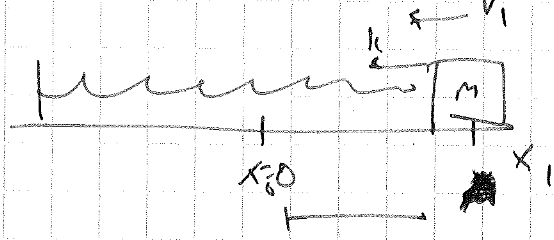
$$\therefore \frac{mv_1^2}{2} - \frac{mv_0^2}{2} = W_{0\to 1}$$

$\frac{mv_1^2}{2} = 0$, $\frac{mv_0^2}{2} = 0$

so: $= -W_{spring}$

$$-W_{spring} = \int_{x_0}^{x_1} (-kx)\, dx = -\frac{k x_1^2}{2} + \frac{k x_0^2}{2}, \quad x_0 = 0$$

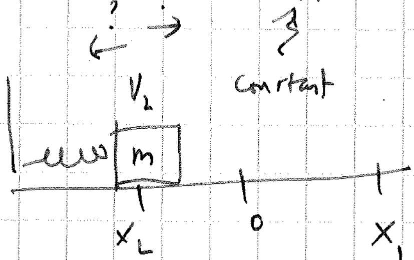
$$\rightarrow \boxed{W_{hand} = \frac{k x_1^2}{2}}$$



E.

$E_{total} =$

constant



$$\frac{mV_2^2}{2} - \frac{mV_1^2}{2} = W_{x_1 \to x_2}$$

sign taken care of

$$\frac{mV_2^2}{2} - \frac{mV_1^2}{2} = \int_{x_1}^{x_2} (-kx)\, dx$$

($dx < 0$ here)

so
$$\frac{mV_2^2}{2} - \frac{mV_1^2}{2} = \left.\frac{-kx^2}{2}\right|_{x_1}^{x_2}$$
$$= -\frac{kx_2^2}{2} + \frac{kx_1^2}{2}$$

$$\Rightarrow \boxed{\frac{mV_2^2}{2} + \frac{kx_2^2}{2} = \frac{kx_1^2}{2} + \frac{mV_1^2}{2}}$$

$\Rightarrow$ so we can define:

PE of spring:

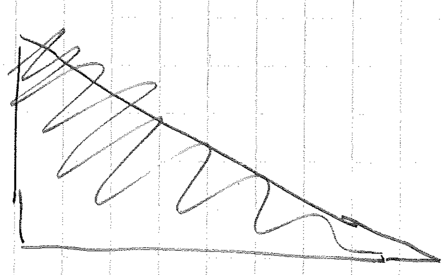
$$U(x) = \frac{kx^2}{2}$$

$x \equiv$ extension displacement

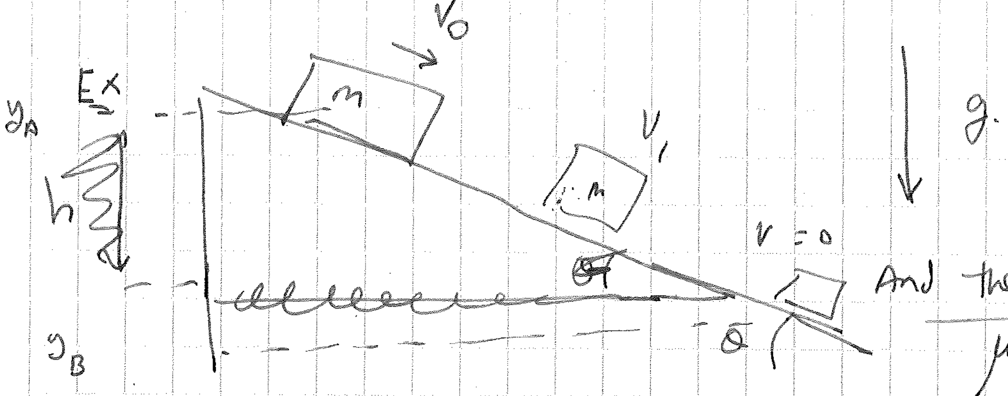
: How

- But it's the difference that matters
- depends only on position of block not how it got there

Ex:



Ex

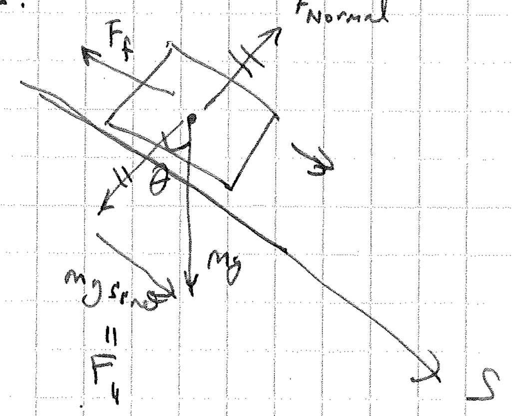


And there's friction μ_k.

Q1: starts with speed v_0. ~~Where does it stops?~~
~~(What's the condition (for it to stop)?)~~
find the speed when it travelled distance ΔS along the slope.

$$W_{A\to B} = -\frac{mv_0^2}{2} + \frac{mv_1^2}{2}$$

Method 1:

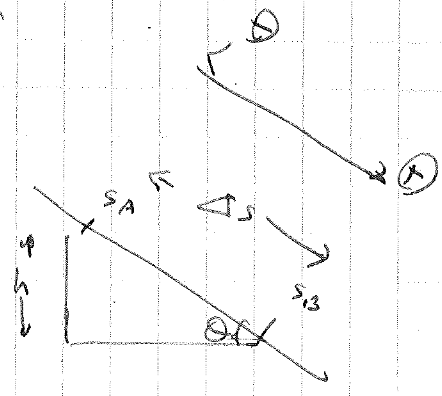


• Reduce to 1-dimensional problem:

$$W_{A\to B} = \int F_{tot}\, dS$$

$$= \int_{s_A}^{s_B} \left(mg\sin\theta - \mu_k mg\cos\theta \right) ds$$

$$= \left(mg\sin\theta - \mu_k mg\cos\theta \right) \Delta S$$



$(S_B > S_A)$

Con:
$$\boxed{\frac{mV_1^2}{2} - \frac{mV_0^2}{2} = mg\left[\sin\theta - \mu_k \cos\theta \right] \Delta S}$$

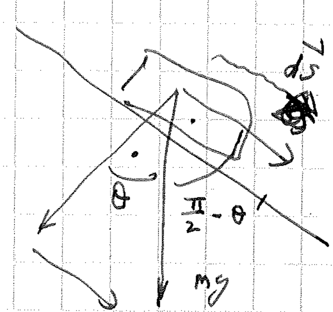
(if $(-)$, then $V_1^2 < V_0^2$)

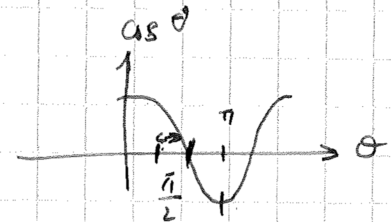
so the thing stops.

~~Math~~ Note Work done by gravity:

$$W_{A\to B}^{\text{gravity}} = mg\sin\theta \,\Delta s$$

$$\vec{F}_{grav}\cdot\vec{ds} = mg\cos\left(\tfrac{\pi}{2}-\theta\right) ds = mg\sin\theta \, ds.$$



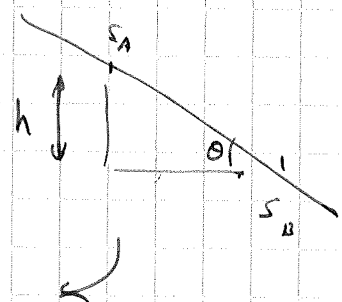


$$\text{So } W_{A\to B}^{\text{grav.}} = \int_A^B \vec{F}_{grav}\cdot d\vec{s}$$

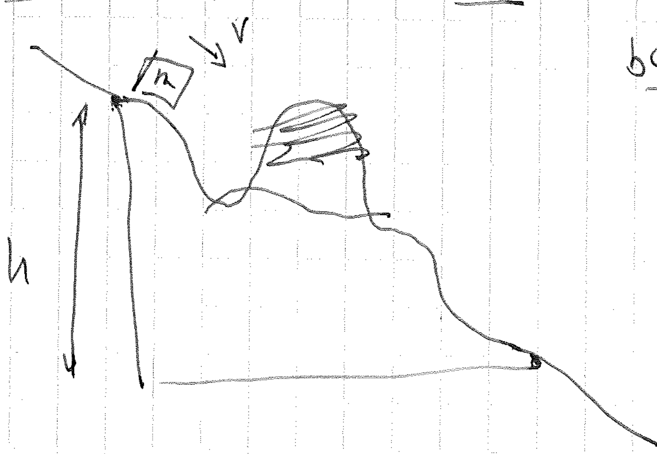
$$= mg\sin\theta \int_{s_A}^{s_B} ds$$

$$= mg\left[\sin\theta\, s_B - \sin\theta\, s_A\right]$$

$$= \boxed{mgh}$$



In fact, this does not depend on path taken.



because

each segment looks like a little ramp.

triangle

Bacteriophage ~~φ~~ φ : Measurements : $W_{total} \sim 10^5 k_B T$

$= \boxed{4.1 \times 10^5\ pN \cdot nm.}$

$0.3\ nm \times 20,000\ bp \approx 6000\ nm.$

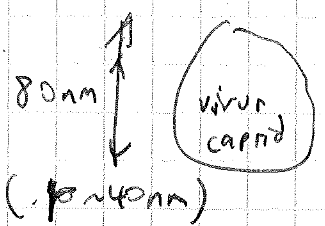
(distance between bases)

$= 6\ \mu m.$

(Note: human DNA (all chromosomes stretched out) $\simeq$ 3 meters long)

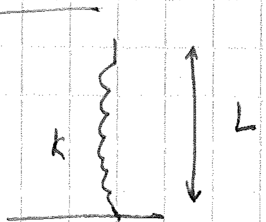
Each chromosome ~ 5 cm long

$6\ \mu m = 6000\ nm. = L$



so $\boxed{\frac{L}{2r} \approx 60}$ so need to compress the DNA 60 times

Simple model :



$$W_{spring} = \frac{k(L_f - L)^2}{2} = \frac{k\left(\frac{L}{60} - L\right)^2}{2}$$

$$= \frac{kL^2}{2}\left[1 - \frac{1}{60}\right]^2$$

$$\sim \boxed{kL^2/2}$$

tiny

$(1 - 1/60 \sim 0.98)$

- Electrostatic W_{charge}:

$+q \xleftrightarrow{\quad r \quad} +q.$ $F_q = \frac{kq^2}{r^2}$

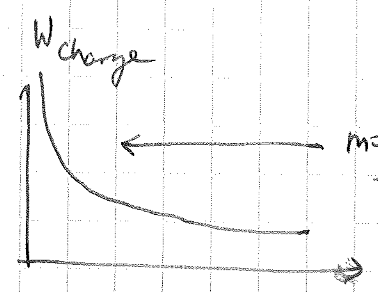
$$\int_r^{r/60} -\frac{kq^2}{r^2}\,dr$$

$$= +\frac{kq^2}{r}\Big|_r^{r/60} = +kq^2\left[\frac{60}{r/60} - \frac{1}{r}\right]$$

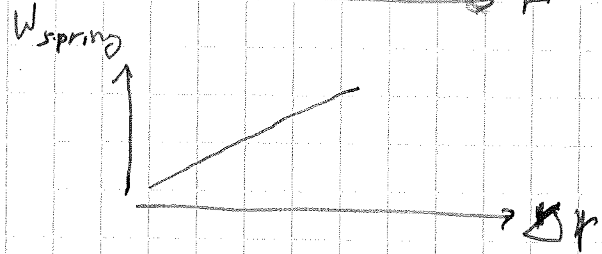
$$= \boxed{+kq^2\,\frac{59}{r}}$$

so if r is very small:

$$W_{charge} \approx k q^2 \frac{59}{r}$$



more "strong"



In fact almost all : electrostatic

$$W_{total} \sim 10^5 k_B T$$
$$= 4.1 \times 10^5 \text{ pN} \cdot \text{nm}$$

↳ Pressure inside capsule $\approx$ 60 atm (1 atm = pressure on ground)

Pressure inside champagne $\approx$ 6 atm.

(10 × champagne)