# Physics 1A for NB

Retake exam (full)

May 4, 2016, 9:00-12:00h

The exam consists of five problems. Make each problem on a separate answer sheet, and hand the sheets in separately. Always show your work, and give full calculations / derivations / arguments. Some equations and physical constants are given on the last page.

## 1 Testing your knowledge (6 points)

NB: When applicable, always explain your answers!

(a) For each of the following laws, indicate under which condition(s) (if any) they're valid: Conservation of energy, conservation of linear momentum, and conservation of angular momentum.

(b) Sketch the position vs. time curve for a block attached to a spring that oscillates back-and-forth as a simple harmonic oscillator without friction. You can assume equilibrium position is at $x = 0$ cm and which is released at $t = 0$ s at $x = 10$ cm with speed zero.

(c) Give the magnitude and direction of the rotation vector of the Earth for the rotation that causes the day-night pattern (i.e. not the slower rotation of Earth around the sun).

(d) Sketch the phase diagram of methane. Indicate all phases and relevant points. Don't forget to label your axes.

## 2 Energy of an electron - 7 points

The potential energy of an electron in a hydrogen atom is given by

$$U(r) = -\frac{a}{r} + \frac{b}{r^2}. \quad (1)$$

Here $a$ and $b$ are positive constants and $r$ is the distance to the origin (where the nucleus of the atom is located).

(a) Give the dimensions of $a$ and $b$.

(b) Does this potential energy produce an attractive or a repulsive force at small distances? And at large distances? (Hint: Sketch the U(r) as a function of $r$)

(c) Find the equilibrium point(s) of this potential energy and determine whether they are stable equilibrium or unstable equilibrium points.

(d) An electron is released at $r = \infty$ with speed zero. Determine the maximum speed the electron can get. (Hint: Total energy of an electron is the kinetic energy plus the potential energy U(r). What is U(r) when r approaches infinity?)

(e) For the electron in (d) that is released from $r = \infty$, find the minimum distance from the nucleus (nucleus is at r=0) that the electron can travel to.

## 3 Mechanics - 8 points

(a) A shell is shot with an initial velocity of 25 m/s, at an angle of 50° with the horizontal. At the top of the trajectory, the shell explodes into two fragments of equal mass. One fragment, whose speed immediately after the explosion is zero, drops to the ground vertically. How far from the gun does the other fragment land (assuming no air drag and level terrain)?

(b) A uniform sphere of radius $R$ is supported by a rope attached to a vertical wall, as shown in figure 1. The rope joins the sphere at a point where a continuation of the rope would intersect a horizontal line through the sphere's center a distance $R/2$ beyond the center, as shown in figure 1. What is the smallest possible value for the coefficient of friction between wall and sphere?

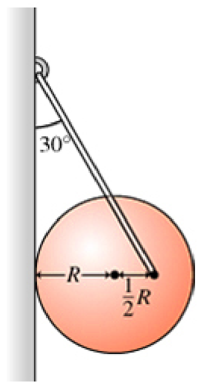


Figure 1: A ball resting on a wall (problem 3b).

(c) A proton (mass 1 u, i.e. mass of one proton) moving at $v_1 = 6.90 \cdot 10^6$ m/s collides elastically and head-on with a second particle moving in the opposite direction at $v_2 = 2.80 \cdot 10^6$ m/s. After the collision, the proton is moving opposite to its initial direction at $8.62 \cdot 10^6$ m/s. Find the mass (in unit of u) and final velocity of the second particle.

# 4 Rotating objects - 8 points

Two children with mass $m_1 = 10$ kg and $m_2 = 10$ kg sit on a simple merry-go-round that can be described as a solid disk of 100 kg with a radius of 2.0 m. The merry-go-round is free to rotate about its center, and initially does so with a frequency $\omega_0$ of 5.0 revolutions per minute. A third child with mass $m_3 = 10$ kg runs towards the merry-go-round with a speed $v_0$ of 1.0 m/s, and under an angle of 30° with the tangent to the merry-go-round (see figure 2a). When arriving at the merry-go-round, the third child jumps on it, and afterwards spins around with the other two.

(a) Find the rotational velocity of the merry-go-round after the third child jumped on it.

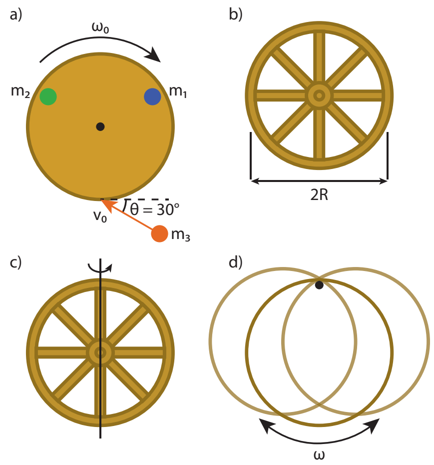


Figure 2: Four rotating systems. a) Three children on a merry-go-round. b) and c) Wagon wheel. d) Hula hoop on a peg.

A wagon wheel is constructed as shown in figure 2b. The radius of the wheel is $R$. Each of the spokes that lie along the diameter has a mass $m$, and the rim has mass $M$ (you may assume the thickness of the rim and spokes are negligible compared to the radius $R$).

(b) What is the moment of inertia of the wheel about an axis through the center, perpendicular to the plane of the wheel?

(c) For the same wheel as in (b), what is the moment of inertia for an axis through the center and two of the spokes, in the plane of the wheel (figure 2c)?

(d) A hula hoop of mass $M$ and radius $R$ hangs from a peg. Find the period of the hoop as it gently rocks back and forth on the peg (figure 2d).

# 5 Thermodynamics - 11 points

We consider an ideal gas with adiabatic exponent $\gamma = 4/3$.

(a) Find the volume specific heat $C_V$ of this gas. Hint: $C_V$ is not $(3/2)R$, since $\gamma \neq 5/3$. Here, you can use the fact that for any ideal gas, the difference between the volume specific heat $C_V$ and the pressure specific heat $C_P$ is $C_P - C_V = R$.

(b) How many atoms are in one molecule of this gas?

We take a sample of this gas that occupies a volume of 5.00 liters, at a temperature of 300 K and a pressure of 100 kPa. The gas is compressed adiabatically to 1/5 of its original volume. Next, its temperature is brought back to 300 K while holding the volume constant. Finally, the gas isothermally expands back to its original volume.

(c) Sketch the pV diagram of the cycle. Indicate any important points, and make sure to put them at the right positions (i.e. calculate symbolically any values of $p$ and $V$, and write down your calculations). Don't forget to properly label your axes. Indicate the direction of the cycle with arrows.

(d) In the second step of the cycle, do you need to cool down or heat up the gas to let it return to its original temperature of 300 K? Explain your answer.

(e) Find the work done on the gas in the entire cycle.

(f) Could this cycle be used as a heat engine? If so, calculate its efficiency. If not, find another application that it could be used for, and calculate its associated coefficient of performance (COP = (what we get out)/(what we put in)).

(g) What should the total change in entropy of the cycle be? Explain your answer.

(h) By directly calculating, find the change in entropy in each of the three steps. (Note: obviously, you can check your answer by summing the contributions of the individual steps to get the total change in (g). However, here you should calculate them explicitly, not use what you know about the total change).

# Some equations and physical constants

| Name | Symbol | Value |
|---|---|---|
| Speed of light | $c$ | $3.00 \cdot 10^8$ m/s |
| Elementary charge | $e$ | $1.60 \cdot 10^{-19}$ C |
| Electron mass | $m_e$ | $9.11 \cdot 10^{-31}$ kg |
| Proton mass | $m_p$ | $1.67 \cdot 10^{-27}$ kg |
| Gravitational constant | $G$ | $6.67 \cdot 10^{-11}$ N $\cdot$ m$^2$/kg$^2$ |
| Gravitational acceleration | $g$ | 9.81 m/s$^2$ |
| Planck's constant | $h$ | $6.63 \cdot 10^{-34}$ J $\cdot$ s |

Table 1: Physical constants

| Object | Moment of inertia |
|---|---|
| Thin rod (length $L$) | $\frac{1}{12}ML^2$ |
| Ring or hollow cylinder (radius $R$) | $MR^2$ |
| Disk or massive cylinder (radius $R$) | $\frac{1}{2}MR^2$ |
| Hollow sphere (radius $R$) | $\frac{2}{3}MR^2$ |
| Massive sphere (radius $R$) | $\frac{2}{5}MR^2$ |
| Rectangle ($a \times b$), perpendicular axis | $\frac{1}{12}M(a^2 + b^2)$ |
| Rectangle ($a \times b$), axis parallel to side with length $b$ | $\frac{1}{12}Ma^2$ |

Table 2: Moments of inertia, all about (symmetry) axes through their center of mass.

| Property | Value |
|---|---|
| Density | 1000 kg/m$^3$ |
| Viscosity | $8.94 \cdot 10^{-4}$ Pa $\cdot$ s |
| Heat of fusion | 334 kJ/kg |
| Heat of vaporization | 2257 kJ/kg |
| Melting point | 273 K |
| Boiling point | 373 K |
| Specific heat (liquid water) | 4184; J/kg $\cdot$ K |
| Specific heat (solid ice) | 2050 J/kg $\cdot$ K |

Table 3: Some properties of water.